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SHOP ARITHMETIC FOR THE MACHINIST

A TREATISE DEALING WITH COMMON MACHINE SHOP AND TOOL-ROOM PROBLEMS AND ARITHMETICAL PROCESSES, WITH NUMEROUS EXAMPLES ILLUSTRATING THEIR PRACTICAL APPLICATION AND METHODS OF SOLUTION

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PREFACE

THIS treatise is intended for the men in machine shops and toolrooms whose training has been chiefly along practical lines, and also for students of mechanical subjects desiring a book featuring the problems commonly encountered in shop work. All subjects have been treated as simply as possible and many examples are used to show exactly how different arithmetical processes are applied to practical shop problems. The advantages and use of formulas are explained and most of the formulas are written out or expressed in words to show the beginner in mathematical studies, just what each formula actually means. In fact, a special effort has been made to explain all methods in plain, simple language.

Many text-books deal with principles rather than with specific examples, but the shop man not used to solving problems, often finds it difficult to apply these principles. The purpose of this treatise, therefore, is to present the most common machine shop and toolroom problems and show clearly, by carefully selected examples, how the principles are applied in each case. It is assumed that the student understands such elementary subjects as addition, subtraction, multiplication, and division of whole numbers and fractions and that he is familiar with the use of common mathematical signs for indicating the different operations referred to. If the student is not entirely familiar with the elementary arithmetical processes mentioned, it is advisable for him to study a text-book on arithmetic before attempting the solution of the problems given in this treatise.

E. O.

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SHOP ARITHMETIC FOR THE MACHINIST

CHAPTER I

FIGURING TAPERS

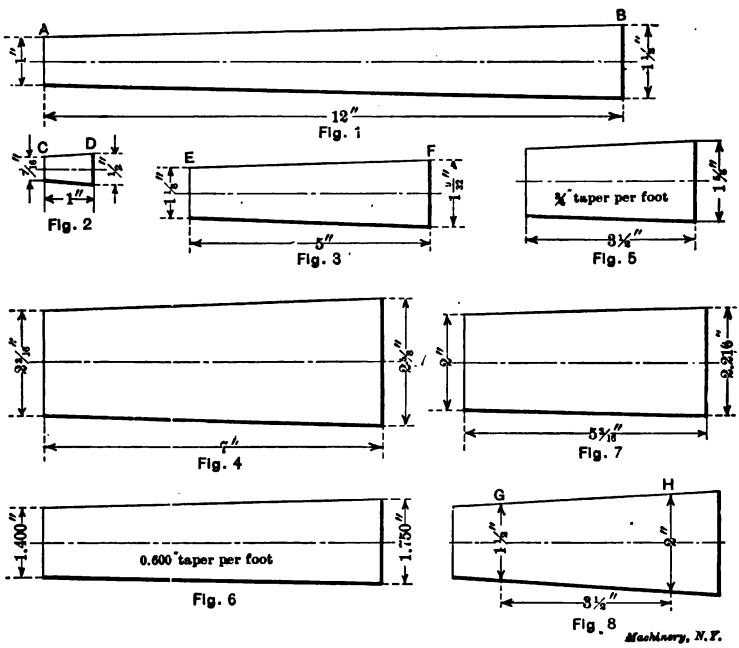
In all cylindrical pieces of work, the expressions "taper per inch" and "taper per foot" mean the taper on the *diameter*, or the difference between the smaller and the larger diameter of a piece, measured one inch or one foot apart, as the case may be. Suppose in Fig. 1 that the diameter at A is one inch, and the diameter at B, one and one-half inch, and that the distance or dimension between A and B is 12 inches or one foot. This piece, then, tapers one-half inch per foot, because the difference between the diameters at A and B is one-half inch. In Fig. 2, the diameter at C is $\frac{7}{16}$ inch, and at D, $\frac{1}{2}$ inch, and the distance between C and D is one inch. This piece, therefore, tapers $\frac{1}{16}$ inch per inch. Tapers may also be expressed for other lengths than one inch and one foot. In Fig. 3, the diameter at E is $1\frac{1}{8}$ inch, and at F, $1\frac{9}{32}$ inch, and the dimension from E to F is 5 inches. This piece of work, therefore, tapers $\frac{5}{32}$ inch in 5 inches, the difference between $1\frac{9}{32}$ and $1\frac{1}{8}$ being $\frac{5}{32}$.

If the taper in a certain number of inches is known, the taper in 1 inch can easily be found. If the taper in 5 inches is $\frac{5}{32}$ inch, the taper in 1 inch equals the taper in 5 inches divided by 5, or, in this case, $\frac{5}{32} \div 5 = \frac{1}{32}$, which is the taper per inch. The taper per foot is found by multiplying the taper per inch by 12. In this case, the taper per foot equals $12 \times \frac{1}{32} = \frac{3}{8}$ inch. The length of the work is always measured parallel to the center line (axis) of the work, and never along the tapered surface.

The problems met with in regard to figuring tapers may be of three classes. In the first place we may have given us the figures for the large and small ends of a piece of work, and the length of the work, as in Fig. 4, and we want to find the taper per foot. In

the second place we may know the diameter at one end, the length of the work, and the taper per foot, as in Fig. 5, and we want to find the diameter at the other end of the work. In the third place we may know the required diameters at both ends of the work and the taper per foot, as in Fig. 6, and we want to find the dimension between the given diameters, or the length of the piece. We will now treat each of these problems in detail.

To Find Taper per Foot when Diameters and Length are Known. Referring to Fig. 4, the diameter at the large end of



Figs. 1 to 8

the work is $2\frac{5}{8}$ inches, the diameter at the small end, $2\frac{3}{16}$ inches, and the length of the work 7 inches. The taper in 7 inches is then equal to the difference between $2\frac{5}{8}$ inches and $2\frac{3}{16}$ inches, or $\frac{7}{16}$ inch. The taper in one inch equals $\frac{7}{16}$ divided by 7, or $\frac{1}{16}$ inch; and the taper per foot is 12 times the taper per inch, or 12 times $\frac{1}{16}$, which equals $\frac{3}{4}$ inch. The taper per foot in Fig. 4, then, equals $\frac{3}{4}$ inch.

If the dimension between the small and the large diameter is not expressed in even inches, but is $5\frac{3}{16}$ inches, for instance, as in

Fig. 7, the procedure is exactly the same. Here the diameter at the large end is 2.216 inches and at the small end 2 inches. The taper in $5\frac{3}{16}$ inches is, therefore, 0.216 inch. This is divided by $5\frac{3}{16}$ to find the taper per inch.

$$0.216 \div 5\tfrac{3}{16} = 0.216 \div \tfrac{83}{16} = 0.216 \times \tfrac{16}{83} = 0.0416$$

The taper per inch consequently equals 0.0416 inch, and the taper per foot is 12 times this amount, or almost exactly $\frac{1}{2}$ inch.

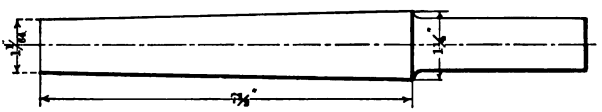


FIG. 9

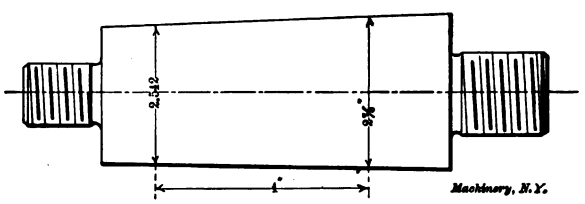


FIG. 10

Figs. 9 and 10

Expressed as a formula, if all dimensions given are in *inches*, the previous calculation would take this form:

$$\text{Taper per foot} = \frac{\text{large dia.} - \text{small dia.}}{\text{length of work}} \times 12$$

It makes, of course, no difference if the large and small diameters are measured at the extreme ends of the work or at some other place on the work, provided the length or distance between the points where the diameters are given, is stated. In Fig. 8, the smaller and larger diameters are given at certain distances from the ends of the work, but the dimension from *G* to *H* is given, and the calculation is exactly the same as if the work were no longer than between *G* and *H*. The following examples will tend to show how the figuring of the taper per foot enters in actual shop work.

Example 1. Fig. 9 shows the blank for a taper reamer. The diameters at the large and small ends of the flutes, and the length

of the fluted part, are stated on the drawing. It is required to find the taper per foot in order to be able to set the taper turning attachment of the lathe.

Referring to the figures given in Fig. 9, the difference in diameters at the large and small ends of the taper is $\frac{15}{64}$ inch. This divided by the length of the flute, $7\frac{1}{2}$ inches, gives us the taper per inch. This we find to be $\frac{1}{32}$. The taper per foot is 12 times the taper per inch, or, in this case, then, $\frac{3}{8}$ inch. The taper attachment of the lathe is, therefore, set to the $\frac{3}{8}$-inch graduation, and the taper turned will be according to the diameters given on the drawing.

Example 2. Fig. 10 shows a taper clamping bolt, entering into the design of a special machine tool. As seen from the cut, the drawing calls for a diameter of $2\frac{7}{8}$ inches a certain distance from the large end of the taper, and for a diameter of 2.542 inches a

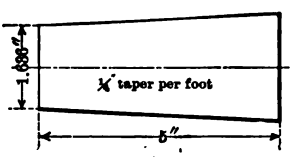


Fig. 11

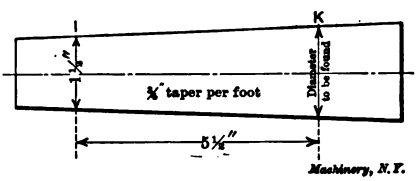


Fig. 12

distance 4 inches farther down on the taper. The taper in 4 inches is then $2\frac{7}{8}$ inches minus 2.542 inches, or 0.333 inch. The taper in one inch equals this divided by 4, or 0.0833. The taper per foot is 12 times the taper per inch, or 12 times 0.0833, which equals one inch, almost exactly. The taper to which to turn the bolt in Fig. 10 is thus one inch per foot.

To Find One Diameter when Taper per Foot, Length and Other Diameter are Known. Referring to Fig. 5, the diameter at the large end of the work is $1\frac{5}{8}$ inch, the length of the work is $3\frac{1}{2}$ inches, and the taper per foot is $\frac{3}{4}$ inch. We now want to find the diameter at the small end. In this case we simply reverse the method employed in our previous problems, where we wanted to find the taper per foot. In this case we know that the taper per foot is equal to $\frac{3}{4}$ inch. The taper in one inch must be one-twelfth of this, or $\frac{3}{4}$ inch divided by 12, which equals $\frac{1}{16}$ inch. Now, the taper in $3\frac{1}{2}$ inches, which we want to find in order to know what the diameter is at the small end of the work, must be $3\frac{1}{2}$ times the

taper in *one* inch, or $3\frac{1}{2}$ times $\frac{1}{16}$, which equals $\frac{7}{32}$. The taper in $3\frac{1}{2}$ inches, then, is $\frac{7}{32}$ inch, which means that the diameter at the small end of a piece of work, $3\frac{1}{2}$ inches long, is $\frac{7}{32}$ inch smaller than the diameter at the large end. The diameter at the large end, according to our drawing, is $1\frac{5}{8}$ inch. The diameter at the small end, being $\frac{7}{32}$ inch smaller, is therefore $1\frac{13}{32}$ inch.

Expressed as a formula, the previous calculation would take this form:

$$\text{Dia. at small end} = \text{dia. at large end} - \left(\frac{\text{taper per foot}}{12} \times \text{length of work}\right)$$

If we now take a case where the diameter at the small end is given, as in Fig. 11, and the diameter at the large end is wanted, the figuring is exactly the same, except of course, we *add* the amount of taper in the length of the work to the small diameter to find the large diameter. When the large diameter is given, we *subtract* the amount of taper in the length of the work to find the small diameter. This is so self-evident that no difficulties ought to be experienced on this account.

Referring again to Fig. 11, where the small diameter is given as 1.636 inch, the length of the work as 5 inches, and the taper per foot as $\frac{1}{4}$ inch, how large is the large diameter of the work? If the taper per foot is $\frac{1}{4}$ inch, the taper per inch is $\frac{1}{4}$ divided by 12 which equals 0.0208, and the taper in 5 inches consequently 5 times 0.0208, or 0.104 inch. The diameter at the large end of the work, which we are figuring, is, then, 0.104 inch larger than the diameter at the small end. The diameter at the small end is given on the drawing as 1.636 inch; adding 0.104 inch to this, we get 1.740 inch as the diameter at the large end.

Expressed as a formula, the previous calculation would take this form:

$$\text{Dia. at large end} = \text{Dia. at small end} + \left(\frac{\text{taper per foot}}{12} \times \text{length of work}\right)$$

It may again be well to call attention to the fact that it makes no difference whether the large and small diameters are figured at the extreme ends of the work or at some other points, as long as the diameter to be found is located at one end of the length dimension, and the diameter stated on the drawing at the other. Thus, in Fig. 12 the diameter stated at *I* is given a certain distance up

on the taper, and the diameter at K, which is wanted, is not at the end of the taper. But the dimension $5\frac{1}{2}$ is given between the points I and K where these diameters are to be measured, and in figuring, one may reason as if the work ended at I and K, the diameter at I being the small diameter, the diameter at K, the large diameter, and $5\frac{1}{2}$ inches the total length of the work. The following examples of direct practical application to shop work will prove helpful in remembering the principles outlined.

Example 1. Fig. 13 shows a taper tap, the blank for which is to be turned. The diameter at the large end of the threaded part is $3\frac{1}{2}$ inches, as given on the drawing, the length of the thread is $6\frac{1}{2}$ inches, and the taper per foot is $\frac{3}{4}$ inch. We want to find the

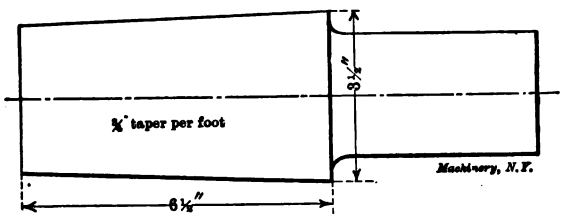


Fig. 13

diameter at the small end, in order to measure this end and ascertain that the tap blank has been correctly turned.

The taper per foot being $\frac{3}{4}$ inch, the taper per inch is $\frac{3}{4}$ divided by 12, or $\frac{1}{16}$ inch. The taper in $6\frac{1}{2}$ inches is $6\frac{1}{2}$ times the taper in one inch, or $6\frac{1}{2}$ times $\frac{1}{16}$ inch, which equals $\frac{13}{32}$ inch. The taper in $6\frac{1}{2}$ inches being $\frac{13}{32}$ inch means that the diameter at the small end of the tap blank is $\frac{13}{32}$ inch smaller than the diameter at the large end. The diameter at the small end is, therefore, $3\frac{3}{32}$ inches.

Example 2. Fig. 14 shows a taper gage for a standard Morse taper No. 1. The diameter at the small end is 0.356 inch, the length of the gage part is $2\frac{3}{8}$ inches, and the taper per foot 0.600 inch. We want the diameter at the large end, in the first place in order to know what size stock to use for the gage, and later for measuring this diameter, when turned, to see that the taper turned is correct.

A taper of 0.600 per foot gives us a taper of 0.050 per inch. In $2\frac{3}{8}$ inches the taper equals $2\frac{3}{8}$ times 0.050, or 0.119 inch. This added to the diameter at the small end gives us the diameter at the large end: $0.356 + 0.119 = 0.475$ inch.

Example 3. Fig. 15 shows a taper bolt used as a clamp bolt. The diameter $3\frac{1}{4}$ inches is given 3 inches from the large end of the taper. The total length of the taper is 10 inches. The taper is $\frac{3}{8}$ inch per foot. We want to find the diameters at the extreme large and small ends of this piece.

We will first find the diameter at the large end. The taper per foot being $\frac{3}{8}$ inch, the taper per inch equals $\frac{1}{32}$ inch. The taper in 3 inches is consequently $\frac{3}{32}$. This added to $3\frac{1}{4}$ inches will give us the diameter at the large end, which is $3\frac{11}{32}$ inches.

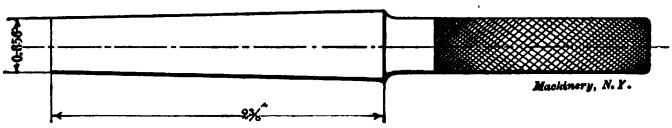


Fig. 14

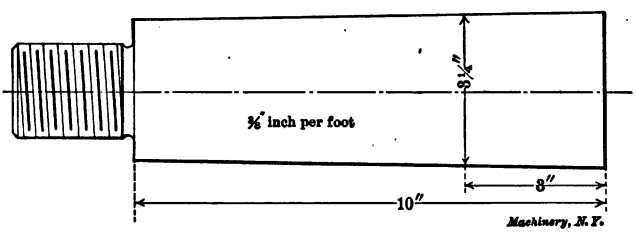


Fig. 15

To find the diameter at the small end, subtract the taper in 10 inches, which is 10 times the taper in one inch, or 10 times $\frac{1}{32}$, which equals $\frac{5}{16}$, from the diameter $3\frac{11}{32}$ inches at the large end. This gives us the diameter at the small end $3\frac{1}{32}$ inches.

We can also find the diameter at the small end without previously finding the diameter at the extreme large end. The total length of the taper is 10 inches, and the dimension from where the diameter $3\frac{1}{4}$ inches is given to the large end is 3 inches. Consequently, the dimension from where the diameter $3\frac{1}{4}$ inches is given to the small end is 7 inches. The taper in one inch was $\frac{1}{32}$ inch; in 7 inches, therefore, $\frac{7}{32}$ inch. The diameter at the small end of the work is $\frac{7}{32}$ inch smaller than $3\frac{1}{4}$ inches, or $3\frac{1}{32}$ inches, the same as found previously when we figured from the extreme large diameter of the taper.

To Find Distance between Two Given Diameters when Taper per Foot is Known. Referring to Fig. 6, if the diameters at both ends of a tapered piece are known, together with the taper per foot, it is required to find the length of the work. Assume that the diameter at the large end of the piece is 1.750 inch, and at the small end, 1.400 inch. The taper per foot is 0.600 inch. How long is this piece of work required to be, in order to have the given diameters at the ends, with the taper stated? We know that the taper per foot is 0.600 inch. The taper per inch is then 0.600 divided by 12, or 0.050 inch. The difference in diameters between the large and the small ends of the work is 1.750 − 1.400, or 0.350 inch, which represents the taper in the length of the work. Now, we know that the taper is 0.050 inch in *one* inch. How many inches does it then require to get a taper of 0.350 inch? This we find by seeing how many times 0.050 is contained in 0.350, or, in other words, by dividing 0.350 by 0.050, which gives us 7 as answer. This means that it takes 7 inches for a piece of work to taper 0.350 inch, if the taper is 0.600 per foot. The length of the work consequently is 7 inches in the case referred to.

Expressed as a formula the previous calculation would take the form:

$$\text{Length of work} = \frac{\text{dia. at large end} - \text{dia. at small end}}{\text{taper per foot} \div 12}$$

The taper per foot divided by 12, as given in the formula above, of course simply represents the taper per inch. The formula may therefore be written:

$$\text{Length of work} = \frac{\text{dia. at large end} - \text{dia. at small end}}{\text{taper per inch}}$$

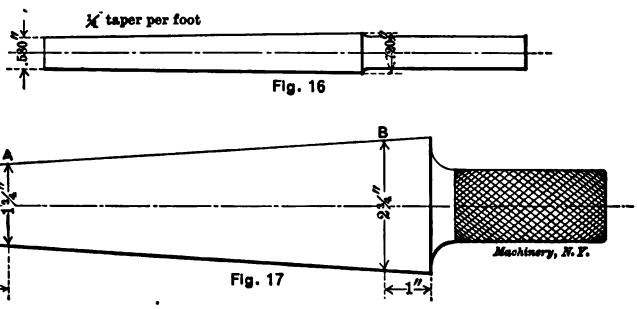
A few examples of the application of these rules will make their use in actual shop work clearer.

Example 1. A taper reamer, Fig. 16, for standard taper pins, having $\frac{1}{4}$ inch taper per foot, is to be made. The diameter at the large end of the flutes is wanted to be 0.720 inch. The diameter at the point of the reamer must be 0.580 inch, in order to accommodate the longest taper pins of this size made. How long should the fluted part of the reamer be made?

The taper per foot is $\frac{1}{4}$ or 0.250 inch, and the taper per inch, consequently, 0.250 divided by 12, or 0.0208 inch. The taper in the length of reamer required is equal to the difference between the

large and the small diameter, or 0.720 − 0.580 equals 0.140 inch. This amount of taper divided by the taper in one inch gives the required length of the flutes. Thus, 0.140, divided by 0.0208 equals 6.731, which represents the length of flutes required. This dimension is nearly $6\frac{3}{4}$ inches, and, being a length dimension of no particular importance, it would be made to an even fractional part of an inch.

Example 2. In Fig. 17 is shown a taper master gage intended for inspecting taper ring gages of various dimensions. The smallest diameter of the smallest ring gage is $1\frac{3}{4}$ inch, and the largest diameter of the largest ring gage is $2\frac{3}{4}$ inches. The taper per foot is $1\frac{1}{2}$ inch. It is required that the master gage extend one inch out-



Figs. 16 and 17

side of the gages at both the small and the large ends, when these are tested. How long should the gage portion of this piece of work be?

The taper per foot is $1\frac{1}{2}$ inch, which is equivalent to $\frac{1}{8}$ inch taper per inch. The total taper from *A* to *B* in Fig. 17 is $2\frac{3}{4}$ minus $1\frac{3}{4}$, or one inch. Therefore, as the taper per inch, $\frac{1}{8}$, is contained in the taper of one inch in the distance from *A* to *B* exactly 8 times, the dimension from *A* to *B* is 8 inches. The gage extends one inch beyond *A* and *B*, respectively, at each end, and the total length of the gage is, therefore, 10 inches.

Rules for Figuring Tapers. 1. If the taper per foot is known, the taper per inch is found by dividing the taper per foot by 12.

2. If the taper per inch is known, the taper per foot is found by multiplying the taper per inch by 12.

3. To find the taper per foot, when the diameters at the large and small ends and the length of the taper are given, subtract

the small diameter from the large, divide the remainder by the length of the taper, and multiply the quotient by 12.

4. To find the diameter at the small end when the diameter at the large end, the length of the taper, and the taper per foot are given, divide the taper per foot by 12, multiply the quotient by the length of the taper, and subtract the resulting dimension from the diameter at the large end.

5. To find the diameter at the large end when the diameter at the small end, the length of the taper, and the taper per foot are given, divide the taper per foot by 12, multiply the quotient by the length of the taper, and add the resulting dimension to the diameter at the small end.

6. To find the dimension between two given diameters of a piece of work, when the taper per foot is given, subtract the diameter at the small end from the diameter at the large end, and divide the remainder by the taper per foot divided by 12.

7. To find how much a piece of work tapers in a certain length, when the taper per foot is given, divide the taper per foot by 12, and multiply the quotient by the dimension of the certain length in which the taper is required.

CHAPTER II

SETTING OVER TAIL-STOCK FOR TAPER TURNING

THE live center and the tail center in a lathe are in alignment when a cutting tool, held in the tool-post of the lathe carriage, traverses in a direction parallel to a line connecting the points of the two centers; if a piece of work is then placed between the centers and revolved, and a cut taken over it, a cylindrical ("straight") piece will be turned. If the tail center is moved out of alignment with the live center an amount A, as shown in Fig. 1, then the center of the work at the tail center end will come nearer to the line of traverse BC of the tool than the center of the work at the live center end, and the diameter of the piece, when turned, will be smaller at the tail center than at the live center. It is therefore a common method to set over the tail-stock when a tapered piece is to be turned. The amount of the taper depends on the length of the work and the "set-over" of the tail center in each case.

When the tail center is set over an amount A, as shown in Fig. 1, the radius (one-half of the diameter) at the small end will be a dimension D smaller than the radius at the large end. This dimension D is also equal to the amount A which the tail center has been set over, and the taper of the work in the length between the centers, therefore, is two times the amount the tail-stock is set over; or, in other words, the tail-stock is set over one-half of the taper in the length of the work.

When Taper per Foot and the Length are Known. The amount which the tail-stock must be set over can be determined if the taper per foot of the work and the length are known.

Assume that a piece of work, $7\frac{1}{2}$ inches long, is required to be turned with a taper per foot equal to $\frac{3}{4}$ inch. We must first know how much the work tapers in $7\frac{1}{2}$ inches. This we find by dividing $\frac{3}{4}$ by 12, and multiplying the quotient by $7\frac{1}{2}$ (see page 10, Rule 7).

$$(\tfrac{3}{4} \div 12) \times 7\tfrac{1}{2} = \tfrac{15}{32}$$

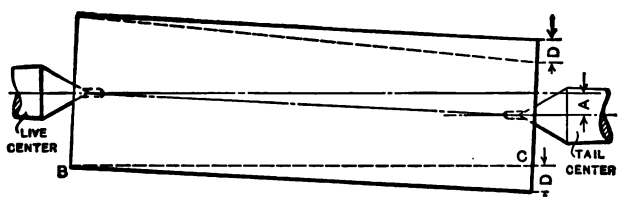
The taper in $7\frac{1}{2}$ inches thus is $\frac{15}{32}$ inch, and as the tail-stock is moved one-half of this, it is set over $\frac{15}{64}$ inch.

When the taper per foot and the length of the work are given, we can calculate the amount to set over the tail-stock from the following formula:

$$\text{Amount to set over tail-stock} = \tfrac{1}{2} \times \left(\frac{\text{taper per foot}}{12} \times \text{length of work}\right)$$

Expressed in words, this formula reads:

To find the amount to set over the tail-stock when the taper per foot and the length of the work are known, divide the taper per foot by 12, multiply the quotient by the length of the work,



and divide the result by 2. (To divide by 2 is the same as to multiply by $\frac{1}{2}$.)

Owing to the fact that the work is not supported by the lathe centers at its extreme ends, but that the lathe centers enter into the work and support it at points a short distance from the ends, it is not practicable to calculate the amount to set over the tail-stock so definitely that the taper can be turned to exact dimensions without a trial cut; but the calculation for setting over the tail-stock gives a close approximation, and when a trial cut on the work has been taken, the final adjustment of the tail-stock to obtain the correct taper can be easily made.

When Diameters at Both Ends of a Tapered Piece are Known. If the diameters at both the large and small ends of work tapering for its full length, are given, the amount to set over the tail-stock can be determined without knowing the taper per foot, because all that is necessary to know is the taper in the length between the centers of the lathe. If, for instance, the diameter at the large end of the work is $1\frac{1}{2}$ inch and the diameter at the small end $1\frac{1}{4}$ inch, as shown in Fig. 2, the amount to set over the tail-stock will

be one-half of the difference between the large and small diameters, or $\frac{1}{8}$ inch. When the diameters at the large and small ends are known, the following formula is therefore used:

$$\text{Amount to set over tail-stock} = \tfrac{1}{2} \times (\text{large diameter} - \text{small diameter})$$

Expressed in words, this formula reads:

To find the amount to set over the tail-stock for work tapering for its full length, when the diameters at the large and small ends are known, subtract the small diameter from the large, and divide the remainder by 2.

When Part of Work is Turned Straight and Part Tapered. If part of the work is turned straight and part of it turned tapered,

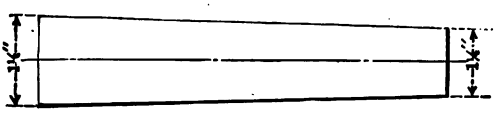


Fig. 2

Fig. 3

as shown in Fig. 3, the taper in the whole length of the work must be determined, and then the tail-stock set over one-half of this amount. In Fig. 3 the work shown is $1\frac{3}{8}$ inch at the small end of the taper. It is tapered for 4 inches, and the diameter at the large end of the taper is $1\frac{5}{8}$ inch. It is then turned straight for the remaining 6 inches, the total length being 10 inches. We must first find what the taper would be in 10 inches if the whole piece had been tapered with the same taper as now required for 4 inches. The taper in 4 inches is $1\frac{5}{8} - 1\frac{3}{8} = \frac{1}{4}$ inch. The taper in 1 inch, consequently, is $\frac{1}{16}$ inch, and in 10 inches, $10 \times \frac{1}{16} = \frac{5}{8}$ inch. The amount to set over the tail-stock is one-half of this, or $\frac{5}{16}$ inch.

If, in a case as shown in Fig. 3, the diameter at the small end is not given, but the taper per foot of the tapered part stated instead, the taper in the total length of the work can be found

directly; if the taper per foot be $\frac{3}{4}$ inch, the taper in 10 inches is $(\frac{3}{4} \div 12) \times 10 = \frac{5}{8}$ inch. (See page 10, Rule 7.) The amount to set over the tail-stock, consequently, is $\frac{5}{16}$ inch. The following formula is used when part of the work is turned straight and part tapered:

$$\text{Amount to set over tail-stock} = \tfrac{1}{2} \times \left(\frac{\text{taper per foot}}{12} \times \text{total length of work}\right)$$

Expressed as a rule, this formula would read:

To find the amount to set over the tail-stock for work partly tapered and partly straight, when the taper per foot and the total length of the work are known divide the taper per foot by 12,

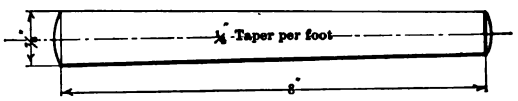


Fig. 4

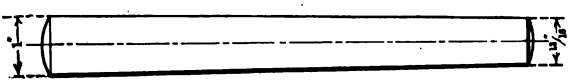


Fig. 5

multiply the quotient thus obtained by the total length of the work, and divide by 2.

If the taper per foot is not given, it must be found before using this formula and rule. (See page 9, Rule 3.)

The following examples will help to give a clear idea of the application of these rules.

Example 1. The taper pin shown in Fig. 4 is 8 inches long, and tapers $\frac{1}{4}$ inch per foot. How much should the tail-stock be set over when turning this pin?

Dividing the taper per foot by 12 gives us 0.0208. Multiplying this figure (which represents the taper per inch) by 8 gives us 0.166 as the taper in 8 inches. Dividing this by 2 gives us the amount required to set over the tail-stock. This amount then is 0.083 inch.

Example 2. Another taper pin, Fig. 5, is 1 inch in diameter at the large end, and $\frac{13}{16}$ inch at the small end. How much should the tail-stock be set over for turning this pin?

The total taper of this pin is found by subtracting the diameter

at the small end, $1\frac{3}{16}$ inch, from the diameter at the large end, 1 inch. This gives us a remainder of $\frac{3}{16}$. One-half of this amount, or $\frac{3}{32}$ inch, represents the amount which the tail-stock should be set over.

Example 3. A taper gage, as shown in Fig. 6, is to be turned by setting over the tail-stock. The diameter at the large end of the taper is $2\frac{1}{4}$ inches, the diameter at the small end is $1\frac{3}{4}$ inch,

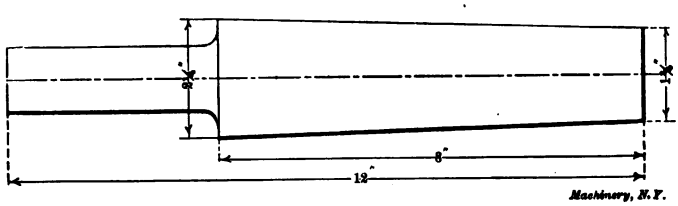


Fig. 6

the length of the taper, 8 inches, and the total length, 12 inches. How much should the tail-stock be set over?

Subtracting the diameter at the small end, $1\frac{3}{4}$ inch, from the diameter at the large end, $2\frac{1}{4}$ inches, gives us a taper of $\frac{1}{2}$ inch in 8 inches. Dividing $\frac{1}{2}$ by 8, gives us the taper in one inch, which is $\frac{1}{16}$ inch. Multiplying this by the total length of the work, 12 inches, gives us $\frac{3}{4}$ inch, which, divided by 2, gives us, finally, the required amount which the tail-stock is to be set over. This latter is, therefore, set over $\frac{3}{8}$ inch.

CHAPTER III

SCREW THREADS AND TAP DRILLS

THE terms pitch and lead of screw threads are often confused. The pitch of a screw thread is the distance from the top of one thread to the top of the next thread, as shown in Fig. 1. No matter whether the screw has a single, double, triple or quadruple thread, the pitch is always the distance from the top of one thread to the top of the next thread. The lead of a screw thread is the distance the nut will move forward on the screw, if it is turned around one full revolution. In the *single-threaded screw, the pitch and lead are equal*, because the nut would move forward the distance from one thread to the next, if turned around once. In a double-threaded screw, however, the nut will move forward two threads, or twice the pitch, so that in a double-threaded screw, the lead equals twice the pitch. In a triple-threaded screw, the lead equals three times the pitch, and so forth. The lead may also be expressed as being the distance from center to center of the *same* thread, after one turn, as indicated in Fig. 2, which shows the pitch and the lead for three screws with Acme threads, the first single-threaded, the second double-threaded, and the last, triple-threaded. In a single-threaded screw, the lead is the distance to the next thread from the one first considered. In a double-threaded screw there are two threads running side by side around the screw, so that the lead is here the distance to the second thread from the one first considered. In a triple-threaded screw, it is the distance to the third thread, and so forth.

The word pitch is often, though improperly, used in the shop to denote *number of threads per inch.* We hear of screws having 12 pitch thread, 16 pitch thread, when 12 threads per inch and 16 threads per inch is what is really meant.

The number of threads per inch is the number of threads counted in the length of one inch, if a scale is held against the side of the screw, and the threads counted as shown in Fig. 3. If there is not a whole number of threads in one inch, count the threads in two or more inches, until the top of one thread comes opposite an

inch-mark, and then divide by the number of inches to find the number of threads in one inch, as shown in Fig. 4.

The number of threads per inch equals 1 divided by the pitch. The pitch of a screw equals 1 divided by the number of threads per inch. Thus, if the number of threads per inch equals 16, the pitch equals $\frac{1}{16}$. If the pitch equals 0.05, the number of

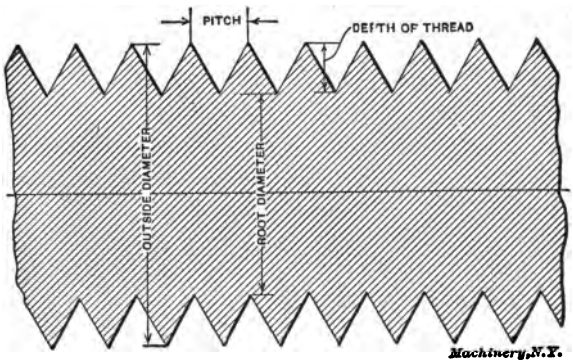


Fig. 1

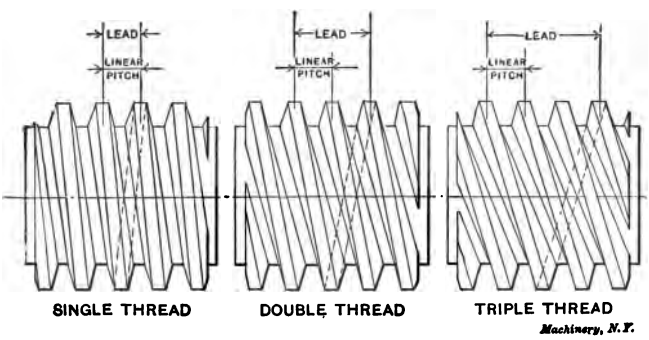


Fig. 2

threads per inch equals $1 \div 0.05 = 20$. If the pitch equals $\frac{2}{5}$ inch, the number of threads per inch equals $1 \div \frac{2}{5} = 2\frac{1}{2}$.

Confusion is often caused by indefinite designation of multiple-thread (double, triple, quadruple, etc.) screws. One way of expressing that a double-thread screw is required is to say, for instance: "3 threads per inch double," which means that the screw is cut with 3 *double* threads, or 6 threads per inch, counting the threads by a scale placed alongside of the screw, as shown in

Fig. 3. The pitch of this screw thus is $\frac{1}{6}$ inch, and the lead twice this, or $\frac{1}{3}$ inch. To cut this screw, the lathe will be geared to cut 3 threads per inch, but the thread will be cut only to the depth required for 6 threads per inch. "Four threads per inch triple" means that there are 4 times 3, or 12 threads along one inch of the screw, when counted by a scale; the pitch of the screw is $\frac{1}{12}$ inch, but being a triple screw, the lead of the thread is 3 times the pitch, or $\frac{1}{4}$ inch.

The best way of expressing that a multiple-thread screw is to be cut, when the lead and the pitch have been figured, is, for example: "$\frac{1}{4}$ inch lead, $\frac{1}{12}$ inch pitch, triple thread." In the case

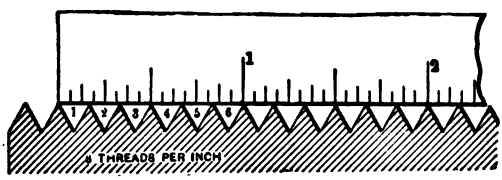


Fig. 3

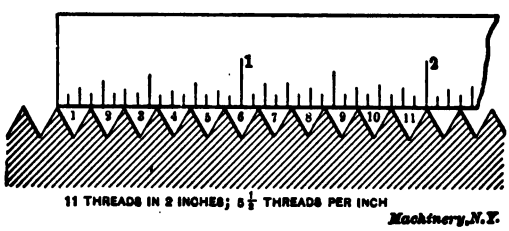


Fig. 4

of single-threaded screws, the number of threads per inch and the form of the thread only are given. The word "single" is not required.

There are three standard threads in common use in American shops. These are shown in Fig. 5, and are the United States standard thread, the sharp V-thread, and the Acme standard thread.

United States Standard Thread. This thread is the most commonly used thread form for all ordinary screws. The thread is provided with a small flat at the top and at the bottom of the thread, as shown in the illustration to the left in Fig. 5.

The depth of the United States (U.S.) standard thread equals

0.6495 × pitch. The width of the flat of the thread at the bottom and top equals $\frac{1}{8}$ × pitch. The root diameter is found by subtracting two times the depth of the thread from the outside diameter of the screw.

[The root diameter of a screw thread is the diameter at the bottom of the thread, as shown in connection with the V-thread in Fig. 1.]

Standard Sharp V-Thread. This thread has no flat at the top or at the bottom, but the sides of the thread form a sharp point, as shown in the illustration, Fig. 5. The depth of the thread equals 0.866 × pitch. The root diameter is found by sub-

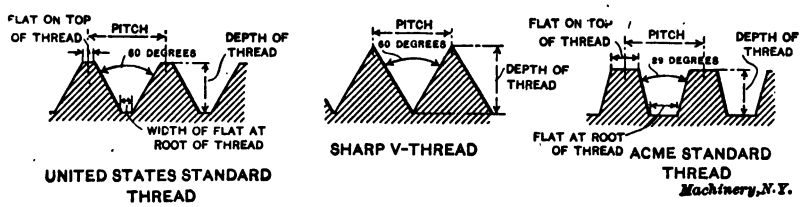


Fig. 5

tracting two times the depth of the thread from the outside diameter of the screw.

Acme Standard Thread. The form of the Acme standard thread is shown to the right in Fig. 5. The depth of the thread equals $\frac{1}{2}$ × pitch + 0.010 inch. The flat at the top of the thread equals 0.3707 × pitch. The width of the flat at the root of the thread equals 0.3707 × pitch − 0.0052 inch. The root diameter of the thread, of course, is found as before, by subtracting two times the depth of the thread from the outside diameter of the screw.

Tap Drill Sizes. The tap drills used for drilling holes previous to tapping are usually somewhat larger in diameter than the root diameter of the thread.

The tap drill diameter for ordinary work for the United States standard thread equals the root diameter + ($\frac{1}{8}$ × pitch).

The tap drill diameter for sharp V-thread equals the root diameter of the thread + ($\frac{1}{4}$ × pitch).

The tap drill diameter for Acme standard thread equals the root diameter + 0.020 inch.

The accompanying table gives the double depths of threads for U. S. and sharp V-threads. The figures in this table opposite any given number of threads per inch are simply subtracted from the outside diameter of the screw, to obtain the root diameter.

Example: Find the root diameter of a $1\frac{5}{16}$ inch diameter screw having 8 U. S. threads per inch. The diameter of the screw in decimals is 1.3125. Subtract from this the double depth of the

TABLE OF DOUBLE DEPTH OF SCREW THREADS

Threads per Inch	U. S. Standard Thread	Standard V-Thread	Threads per Inch	U. S. Standard Thread	Standard V-Thread
$2\frac{1}{4}$	0.5774	0.7698	18	0.0722	0.0962
$2\frac{3}{8}$	0.5470	0.7293	20	0.0650	0.0866
$2\frac{1}{2}$	0.5196	0.6928	22	0.0590	0.0787
$2\frac{5}{8}$	0.4949	0.6598	24	0.0541	0.0722
$2\frac{3}{4}$	0.4724	0.6298	26	0.0500	0.0666
$2\frac{7}{8}$	0.4518	0.6025	28	0.0464	0.0619
3	0.4330	0.5774	30	0.0433	0.0577
$3\frac{1}{4}$	0.3997	0.5329	32	0.0406	0.0541
$3\frac{1}{2}$	0.3712	0.4949	34	0.0382	0.0509
4	0.3248	0.4330	36	0.0361	0.0481
$4\frac{1}{2}$	0.2887	0.3849	38	0.0342	0.0456
5	0.2598	0.3464	40	0.0325	0.0433
$5\frac{1}{2}$	0.2362	0.3149	42	0.0309	0.0412
6	0.2165	0.2887	44	0.0295	0.0394
7	0.1856	0.2474	46	0.0282	0.0377
8	0.1624	0.2165	48	0.0271	0.0361
9	0.1443	0.1925	50	0.0260	0.0346
10	0.1299	0.1732	52	0.0250	0.0333
11	0.1181	0.1575	56	0.0232	0.0309
12	0.1083	0.1443	60	0.0217	0.0289
13	0.0999	0.1332	64	0.0203	0.0271
14	0.0928	0.1237	68	0.0191	0.0255
15	0.0866	0.1155	72	0.0180	0.0241
16	0.0812	0.1083	80	0.0162	0.0217

These constants are subtracted from the outside diameter of the tap or screw; the result is the root diameter of the thread

thread, or the so-called "constant," in the table, which is 0.1624, as given opposite 8 threads per inch. The remainder, 1.1501, is the root diameter of the thread.

When the table of double depth of threads is used for multiple-thread screws, it should be noted that the number of threads per inch as measured along the screw as shown in Fig. 3, should be taken. Thus, for instance, to find the root diameter of a screw having 6 threads per inch double, the figures opposite 12 threads (6 double) should be used.

CHAPTER IV

SPEEDS OF GEARS AND PULLEYS

SUPPOSE that two shafts *A* and *B*, as shown in Fig. 1, are to be connected by gearing so that shaft *A* makes one revolution while shaft *B* makes three. To obtain this result, the gear on *A* must have three times as many teeth as the gear on *B*. Assume that we have 90 teeth in the gear on *A*; the gear on *B* must then have only 30 teeth. Each time the small gear on *B* turns around one complete revolution, it engages 30 teeth in the gear on shaft *A*. It therefore must turn around three full revolutions in order to engage all the 90 teeth in the larger gear, or, in other words, the smaller gear revolves three times in order to revolve the gear on *A* once. The *ratio* of this gearing is 3 to 1. If one gear revolves three times while the mating gear revolves four times, the ratio of the gearing would be 3 to 4. The ratio of gearing expresses the relation between the number of times which one gear revolves and the number of times the mating gear revolves in the same time.

Effect of Idlers. Assume that the shafts *C* and *D* in Fig. 2 are required to run in a ratio of 5 to 1, that is, shaft *C* is to revolve five times while shaft *D* revolves once. If a 20-tooth gear is placed on shaft *C*, the gear on shaft *D* must have five times as many teeth, or 100 teeth; then it will revolve but once when the gear on *C* revolves five times. Assume that we put gears with the number of teeth mentioned on the shafts. The distance between the shafts may now be such that the gears will not mesh or engage. An intermediate gear *E* mounted on a stud *F* may then be placed so that it meshes with both the gear on *C* and the gear on *D*. The intermediate gear simply transmits motion from the gear on *C* to the gear on *D*, but has no influence on the ratio of speed of the shafts *C* and *D*. When the intermediate gear is in place, the gear on *C* still revolves five times while the gear on *D* revolves once.

If we place a number of intermediate gears, *E*, *F*, and *G*, in the train, as in Fig. 4, the result would still be the same, the gear on *C* would turn 5 times while the gear on *D* turned once, as long as the

number of teeth in the gear on D is 5 times the number of teeth in the gear on C.

In order to prove this, let us assume that in Fig. 4, the gear on stud C has 20 teeth, and the gear on stud D, 100 teeth, so that consequently the stud C makes 5 revolutions, while stud D makes one. The intermediate gears, E, F, and G, have 50, 40, and 40 teeth, respectively, as shown in the cut. Now, when the gear on D turns around once, the gear G must turn $2\frac{1}{2}$ times ($\frac{100}{40} = 2\frac{1}{2}$).

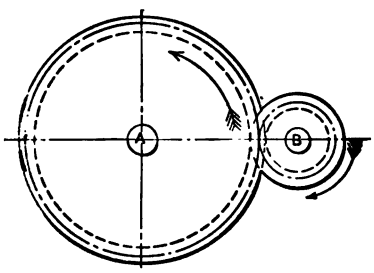


Fig. 1

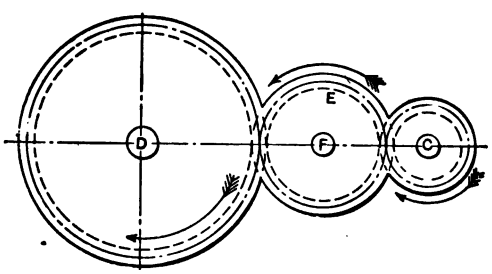


Fig. 2

The gear F, having the same number of teeth as gear G, makes one revolution while G makes one, and consequently also turns $2\frac{1}{2}$ times while the gear on D turns once. The gear on E, having 50 teeth, turns $\frac{4}{5}$ of a revolution while gear F revolves once ($\frac{40}{50} = \frac{4}{5}$), and consequently, while F makes $2\frac{1}{2}$ revolutions, gear E makes $2\frac{1}{2} \times \frac{4}{5} = \frac{5}{2} \times \frac{4}{5} = 2$ revolutions. Thus E turns twice while the gear on stud D revolves once. Finally, the gear on C turns $2\frac{1}{2}$ times to each revolution of gear E ($\frac{50}{20} = 2\frac{1}{2}$), or 5 times to 2 revolutions of E. But 2 revolutions of E correspond, as we have seen, to one revolution of the gear on stud D; consequently, the gear on stud C makes 5 revolutions to one of the gear on stud D,

which, as we previously said, is also the case if these two gears are connected directly without any intermediate gearing.

The intermediate gears, however, affect the direction in which the gear on *D* revolves. In Fig. 1, when the gear on *B* revolves

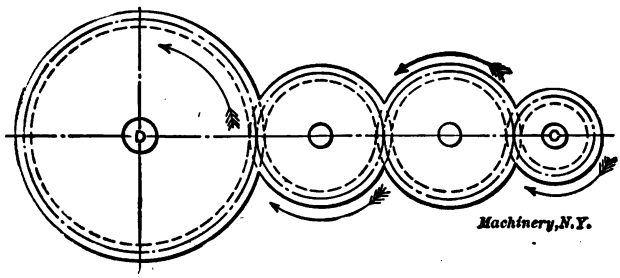


Fig. 3

in a right-hand direction (in the same direction as the hands of a watch), the gear on *A*, which is driven by it, will move in a left-hand direction (in a direction opposite to that of the hands of a watch). Thus when there is no intermediate gear, the driving gear

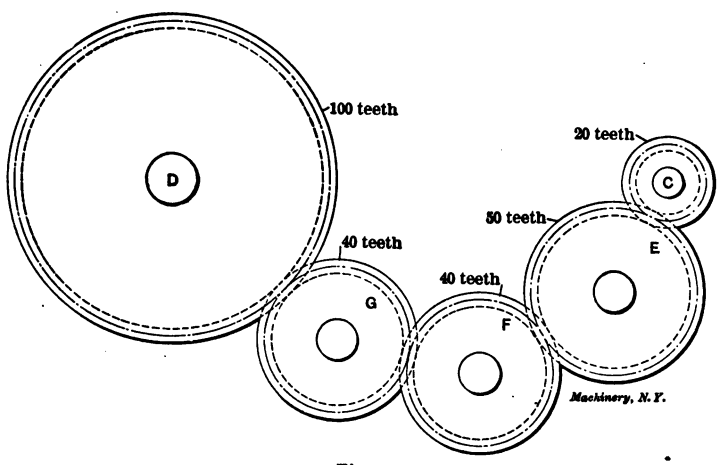


Fig. 4

and the driven gear revolve in opposite directions. In Fig. 2, again, the gear on *C* moves in a right-hand direction, the gear on *F* in a left-hand direction, and the gear on *D* in a right-hand direction, so that in this case both the driver on *C* and the driven gear on *D* move in the same direction.

If there be two idlers, as shown in Fig. 3, the speed ratio between the shafts C and D still remains the same as before, but it will be seen from the arrows indicating the directions in which the different gears revolve, that in this case, the driver on C and the driven gear on D move in opposite directions. The use of two idlers, therefore, makes the driven gear run in the same direction as if there was no idler between the gears on C and D. If there were three idlers the gears on C and D would run in the same direction,

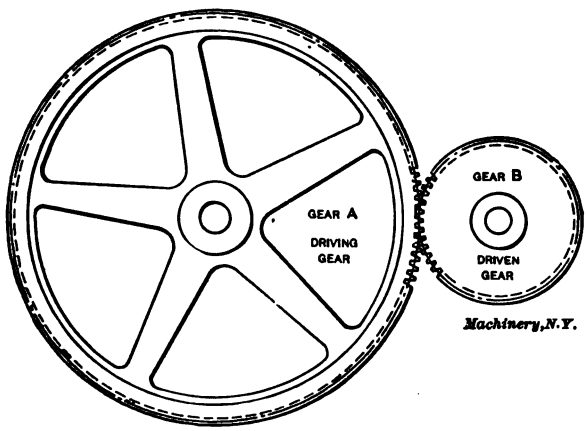


Fig. 5

and if there were four idlers, they would run in opposite directions, and so on.

Gears Required for a Given Speed Ratio. If we have two gears A and B, as shown in Fig. 5, and the ratio of the speed of gear A to the speed of gear B and the number of teeth in one of the gears are given, the number of teeth required in the other gear can be determined; and if the ratio only is given, the number of teeth in both the gears can be found.

Assume that the ratio of the speed of gear A to gear B is 1 to 4. This means that gear A revolves once while gear B revolves four times. If there be 80 teeth in gear A, gear B must have one-fourth of this number, or 20 teeth. Had the speed ratio been 2 to 5, then if gear A had 50 teeth, gear B would have $\frac{2}{5} \times 50 = 20$ teeth. From this we may formulate the following rule:

If the speed ratio of the driving gear to the driven gear and the number of teeth in the driving gear are given, the number of teeth

in the driven gear may be found by multiplying the number of teeth in the driving gear by the speed ratio, written as a fraction.

Assume, for instance, that the speed ratio of gear A to gear B is 3 to 5, and that the number of teeth in gear A is 60. Writing the speed ratio as a fraction gives us $\frac{3}{5}$, and this multiplied by 60 gives us 36, which is the number of teeth in the gear B.

Note that when the speed ratio of gear A to gear B is given, we multiply the number of teeth in gear A with the speed ratio written as a fraction, to get the number of teeth in gear B. But if the number of teeth in gear B is given, and the number of teeth in gear A is to be found, the number of teeth in gear B should be multiplied by the *inverted* speed ratio. (To invert a fraction means to turn it upside down, so that the numerator becomes the denominator, and the denominator becomes the numerator.) If the speed ratio of gear A to gear B is 2 to 3, and the number of teeth in gear B is 34, then the number of teeth in gear A is found by multiplying 34 by the inverted ratio; thus:

$$34 \times \tfrac{3}{2} = \tfrac{102}{2} = 51$$

It is helpful to remember when doing calculations of this kind that the larger gear, which, of course, is the gear with the greater number of teeth, is always the gear which will make the fewer number of revolutions. Keeping this in mind will prevent mistakes.

If the speed ratio of two shafts is given, and the numbers of teeth for two gears which will transmit motion between the shafts at the given speed ratio are to be found, write the speed ratio in the form of a fraction and multiply the numerator and the denominator by the same number until a new fraction is obtained, having numerator and denominator expressing suitable numbers of teeth for the gears.

If the given speed ratio be 4 to 7, and it is required to find two gears which will transmit motion between the two shafts at this ratio, write the ratio as a fraction ($\frac{4}{7}$), and multiply numerator and denominator by the *same* number, thus:

$$\frac{4}{7} = \frac{4 \times 8}{7 \times 8} = \frac{32}{56}$$

By multiplying 4 and 7 by 8 we obtain two gears with 32 and 56 teeth, which will transmit the motion from one shaft to another

at the required ratio. The larger gear, with 56 teeth, is placed on the shaft with the lowest number of revolutions.

Finding the Number of Revolutions per Minute. If the numbers of teeth in the two gears and the number of revolutions per minute of one shaft are known, and it is required to find the number of revolutions of the other shaft, the following rule is used:

The number of revolutions of one shaft multiplied by the number of teeth of the gear on the same shaft, divided by the number of teeth in the gear on the second shaft, gives the number of revolutions of the second shaft.

Written as a formula this rule would be:

$$\frac{\text{Rev. of first shaft} \times \text{teeth in gear on first shaft}}{\text{teeth in gear on second shaft}} = \text{rev. of second shaft}$$

If the speed ratio is given instead of the numbers of teeth, the numbers of teeth may first be found by the rules previously given.

Finding the Speed Ratio. The speed ratio can be found if either the numbers of teeth in both of the gears, or the numbers of revolutions per minute of both of the gears, are given.

If the numbers of revolutions of two gears A and B are given, the ratio of the speed of gear A to gear B is found by placing the number of revolutions of gear A as the numerator and the revolutions of gear B as the denominator of a fraction, and reducing this fraction to its lowest terms. If gear A makes 34 revolutions, and gear B, 85 revolutions, the speed ratio of gear A to gear $B = \frac{34}{85} = \frac{2}{5}$ or, as it is commonly expressed, 2 to 5.

If the numbers of teeth in the gears A and B are given, the speed ratio of gear A to gear B is found by writing the number of teeth of gear B as the numerator, and of gear A as the denominator of a fraction, and reducing the fraction to its lowest terms. If the number of teeth in gear A is 60, and in gear B, 48, the ratio of speed of gear A to gear $B = \frac{48}{60} = \frac{4}{5}$, or 4 to 5.

Compound Gearing. The simplest and most common case of *compound* gearing consists of four gears mounted as shown in Fig. 6. The gear A is the driver and meshes with gear B, which is keyed to the same shaft as gear C. Thus, when gear B is driven by gear A, gear C will revolve at the same rate of speed as gear B, and will drive gear D; the motion of gear A is thus transmitted to gear D. The two gears B and C which serve to transmit the

motion are called *intermediate* gears. The gears *A* and *C* are the *driving* gears, and *B* and *D* the *driven* gears.

If it is required to find the revolutions of the driven gear *D* when the number of revolutions of gear *A* and the numbers of teeth in all the gears are known, the following rule is used:

The revolutions per minute of the driven gear equal the revolutions per minute of the driving gear, times a fraction, the numerator of which is made up of the product of the numbers of teeth in

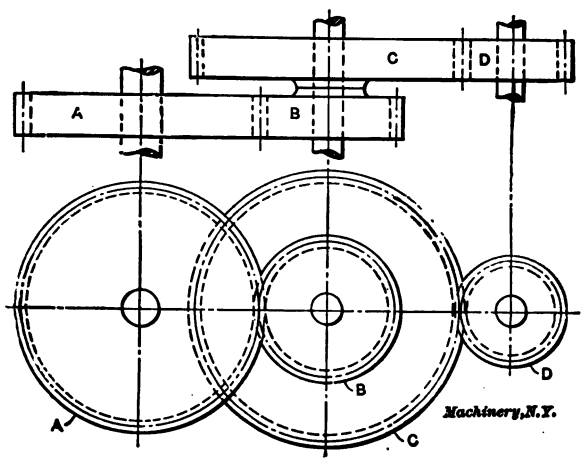


Fig. 6

the driving gears, and the denominator of the product of the numbers of teeth in the driven gears.

This rule can be written as a formula as below:

$$\text{Rev. per min. of driven gear} = \text{rev. per min. of driving gear} \times \frac{\text{product of teeth in driving gears}}{\text{product of teeth in driven gears}}$$

This formula can be used whether there be one or more sets of intermediate gears.

Assume that gear *A* in Fig. 6 has 40 teeth, gear *B*, 24 teeth, gear *C*, 50 teeth, and gear *D*, 25 teeth. Then if gear *A* makes 30 revolutions per minute, how many revolutions does gear *D* make? Using the formula just given we have:

$$\text{Rev. per min.} = 30 \times \frac{40 \times 50}{24 \times 25} = 100$$

The revolutions per minute made by gear B may be found from the same formula by leaving out the numbers of teeth of gears C and D, thus:

$$\text{Rev. per min.} = 30 \times \tfrac{4}{2}\tfrac{0}{4} = 50$$

Finding Number of Teeth in Gears to Transmit Motion at a Given Ratio. If the numbers of revolutions of the driving gear A and the driven gear D are given, the numbers of teeth required in the four gears of a compound gearing that will transmit motion at the required ratio, can be found.

Assume that gear A, Fig. 6, makes 36 revolutions per minute and that it is required that gear D should make 56 revolutions. The speed ratio is then $\frac{3}{5}\frac{6}{6} = \frac{9}{14}$. (See page 26.)

To find the gears required, write the ratio of the speed of the driving gear to the driven gear as a fraction, divide the numerator and denominator in two factors, and multiply each "pair" of factors by the same number until gears with suitable numbers of teeth are found. (One factor in the numerator and one in the denominator make "one pair.") In this example

$$\frac{9}{14} = \frac{3 \times 3}{2 \times 7} = \frac{(3 \times 20) \times (3 \times 10)}{(2 \times 20) \times (7 \times 10)} = \frac{60 \times 30}{40 \times 70}$$

The gears in the numerator, with 60 and 30 teeth, are the driven gears (gears B and D, Fig. 6), and the gears in the denominator, with 40 and 70 teeth, are the driving gears (gears A and C, Fig. 6).

The calculation may be expressed in a formula as follows:

$$\text{Ratio of speed of the first driving gear to the last driven gear} = \frac{\text{product of teeth in driven gears}}{\text{product of teeth in driving gears}}$$

Speed of Pulleys. The principle applied to gearing in regard to the ratio between the speeds of two shafts, may be directly applied to the question of sizes of pulleys, with the only difference that we here deal with the number of inches to the diameter of the pulley instead of the number of teeth in the gear.

Assume that a shaft is required to make 300 revolutions per minute, and that it is driven from a line-shaft making 180 revolutions per minute, as shown in Fig. 7. The pulley on the line-shaft is in place, and is 15 inches in diameter. What diameter should the pulley on the shaft making 300 revolutions per minute be made? As the belt on the two pulleys runs at the same speed as

the periphery (circumference) of either of the pulleys, it is clear that the peripheries of both pulleys run at the same speed, provided there is no slippage between the belt and the pulleys. The pulley running a smaller number of revolutions must, of course, be larger in order that its periphery may run at the same speed as the periphery of the pulley making a greater number of revolutions. The circumference of a circle (and, therefore, also the circumference of a pulley) equals the diameter $\times$ 3.1416. Therefore, the circumference of the pulley making 180 revolutions and having a diameter of 15 inches, passes, in one minute, through a distance equal to 180 times its circumference, or $180 \times 15 \times 3.1416$.

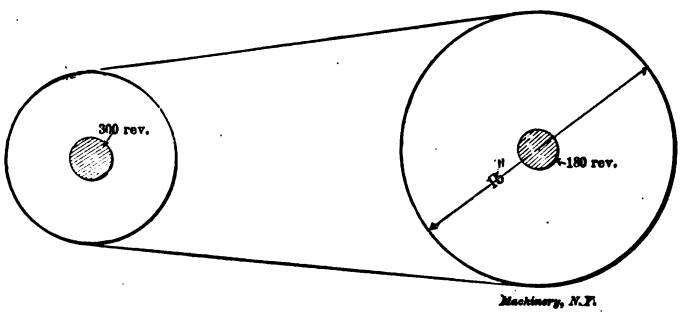


Fig. 7

The circumference of the pulley making 300 revolutions must pass through the same distance in one minute; therefore, for *each* revolution this pulley must pass through the distance $180 \times 15 \times 3.1416$ divided by 300. This, then, would equal the circumference of the smaller pulley; but the circumference also equals the *diameter* $\times$ 3.1416. We can therefore write:

$$\frac{180 \times 15 \times 3.1416}{300} = \text{diam. of smaller pulley} \times 3.1416$$

As 3.1416 enters as a factor on both sides of the equals sign, it can be cancelled. Then we have:

$$\frac{180 \times 15}{300} = \text{diameter of smaller pulley}$$

From this we can formulate the following rule for the relation between the sizes of pulleys and the number of revolutions of two shafts:

The number of revolutions of one shaft multiplied by the diameter of the pulley on the same shaft divided by the number of revolutions of the second shaft, gives the diameter of the pulley on the second shaft.

We can write this rule in the form of a formula, as follows:

$$\text{Diam. of pulley on second shaft in inches} = \frac{\text{no. of rev. of first shaft} \times \text{diam. of pulley on first shaft in inches}}{\text{no. of rev. of second shaft}}$$

If one pulley makes 200 revolutions while another pulley makes 100 revolutions, we say that the *speed ratio* between the two pulleys is 2 to 1. If one pulley makes 200 revolutions while another makes 50 revolutions, the speed ratio is 4 to 1, because one shaft makes four times as many revolutions as the other. If one shaft runs 200 revolutions and another 140 revolutions, the speed ratio would be 200 to 140. By cancelling equal factors in 200 and 140, we can reduce the ratio so as to express it with smaller numbers. In this case the ratio would be 10 to 7; ($\frac{200}{140} = \frac{20}{14} = \frac{10}{7}$).

[If very accurate results are required, one thickness of the belt should be added to the diameter of the pulley itself, and the dimension thus obtained should be used in the formulas above instead of the diameter of the pulley rim. For instance, if the pulley is 5 inches in diameter and the belt $\frac{1}{8}$ inch thick, the diameter to be used in the formulas should be $5\frac{1}{8}$ inches. The results obtained in this manner will be very accurate provided there is no slipping of the belt on either of the pulleys. For ordinary practical purposes, however, it is usual to figure with the diameter of the pulley rim, taking no account of the thickness of the belt.]

Finding the Number of Revolutions of a Shaft. If we know the diameters of the pulleys and the number of revolutions per minute of one shaft, and want to find the number of revolutions of the other shaft, we use the following rule:

The number of revolutions on one shaft multiplied by the diameter of the pulley on the same shaft, divided by the diameter of the pulley on the second shaft, gives the number of revolutions of the second shaft.

This rule can be written as a formula as follows:

$$\text{No. of rev. of second shaft} = \frac{\text{no. of rev. of first shaft} \times \text{diam. of pulley on first shaft}}{\text{diam. of pulley on second shaft}}$$

Practical Applications. *Example* 1. A lineshaft for some grinding machines is required to run at 320 revolutions per minute and is to be driven from a main lineshaft running at 200 revolutions per minute. The pulley on the main lineshaft is already in place and is 24 inches in diameter. What diameter ought the pulley of the grinding machine lineshaft to be? If we apply our first rule or formula we have directly:

$$\frac{200 \times 24}{320} = 15$$

The diameter of the pulley on the grinding machine lineshaft therefore should be 15 inches.

Example 2. The pulley on a geared-head lathe is 16 inches in diameter. The driving pulley on the lineshaft is 34 inches in diameter, and the lineshaft makes 120 revolutions per minute. How many revolutions does the 16-inch pulley on the lathe make?

If we apply the second rule or formula given we have directly:

$$\frac{120 \times 34}{16} = 255$$

The 16-inch pulley consequently makes 255 revolutions per minute.

Example 3. The largest step of a cone-pulley on the countershaft of a machine is 12 inches in diameter. The smallest step is 6 inches. The largest step on the cone-pulley on the machine is 10 inches, the smallest, 4. If the countershaft runs at 300 revolutions per minute, what are the highest and lowest speeds of the spindle on which the cone-pulley on the machine is mounted?

The largest step of each respective pulley runs with the smallest step of the other. We can, therefore, proceed as if we had two sets of pulleys, one set 12 and 4, and one, 6 and 10 inches in diameter, and by using the second rule or formula given we have:

$$\frac{300 \times 12}{4} = 900 \quad \text{and} \quad \frac{300 \times 6}{10} = 180$$

The highest speed is therefore 900 revolutions per minute and the lowest 180 revolutions.

Compound Pulley Speeds. In Fig. 8 are shown four pulleys of which the two pulleys B and C are keyed to the same shaft. Pulley A is the driving pulley and drives pulley B; pulley C, on the same shaft as B, is also a driving pulley, and pulley D, a driven

pulley. The rules and formulas for compound gearing can be directly applied to pulleys arranged in this manner by simply substituting in the formulas the diameters of the pulleys, in inches, for the numbers of teeth in the gears. Thus, to find the revolutions per minute of the driven pulley D when the diameters of all the four pulleys and the number of revolutions of pulley A are given, the formula below is used:

$$\text{Rev. per min. of driven pulley} = \text{rev. per min. of driving pulley} \times \frac{\text{product of diam. of driving pulleys}}{\text{product of diameters of driven pulleys}}$$

If the numbers of revolutions of the shafts on which pulley A and pulley D are mounted, are given, and it is required to find the

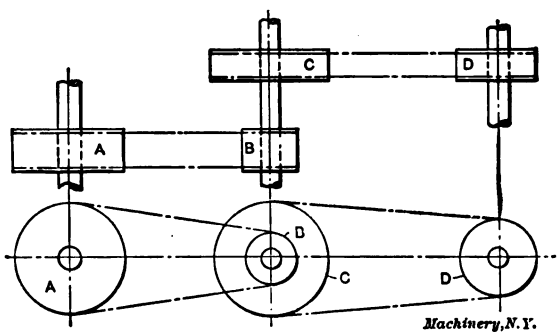


Fig. 8. Compound Pulley Drive

diameters of four pulleys which will transmit motion from pulley A to pulley D at the given speed ratio, we proceed in the same way as when finding the number of teeth in gears for transmitting a given motion.

Find the speed ratio by writing the number of revolutions of the driving pulley as the numerator and the number of revolutions in the driven pulley as the denominator of a fraction, and reduce this fraction to its lowest terms. Then divide both the numerator and denominator in the fraction giving the ratio in two factors, and multiply each "pair" of factors by the same number until pulleys with suitable diameters are found. (One factor in the numerator and one in the denominator are considered as "one pair.")

Assume that the number of revolutions per minute of the shaft with pulley A is 260, and that it is required to drive the shaft on

which pulley D is mounted at 720 revolutions. What diameters of pulleys can be used? The fraction $\frac{360}{720}$ reduced to its lowest terms is $\frac{13}{36}$; ($\frac{360}{720} = \frac{36}{72} = \frac{13}{36}$). The speed ratio, therefore, is $\frac{13}{36}$. Now, following the rule given above:

$$\frac{13}{36} = \frac{1 \times 13}{2 \times 18} = \frac{(1 \times 12) \times (13 \times 1)}{(2 \times 12) \times (18 \times 1)} = \frac{12 \times 13}{24 \times 18}$$

The pulleys in the numerator, with 12 and 13 inches diameter, are the driven pulleys B and D, and the pulleys in the denominator,

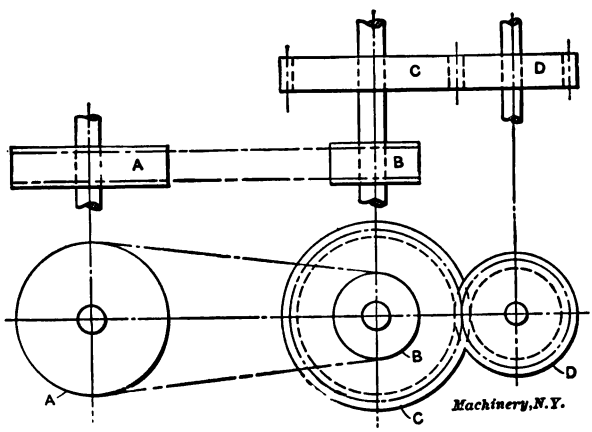


Fig. 9. Combined Pulley and Gear Drive

with 18 and 24 inches diameter, are the driving pulleys. This rule reduced to a formula would be:

$$\text{Ratio of speed of the first driving pulley to the last driven pulley} = \frac{\text{product of diam. of driven pulleys}}{\text{product of diam. of driving pulleys}}$$

Combined Belt and Gear Drive. In Fig. 9 is shown a combined belt and gear drive, where pulley A drives pulley B, and gear C, which is mounted on the same shaft as pulley B, drives the gear D. Calculations for numbers of revolutions and numbers of teeth and diameters of pulleys are carried out exactly as in the examples where we have dealt exclusively with gears or exclusively with pulleys. When dealing with the pulleys we use the diameter of the pulley in inches, and when dealing with the gears, the number of teeth in the gears.

Assume that the diameter of pulley A is 54 inches and that of pulley B, 18 inches, that gear C has 112 teeth, and that gear D has 78 teeth. If pulley A makes 39 revolutions per minute, how many revolutions per minute does gear D make? Using the formula for finding the revolutions per minute previously given, we have:

$$\text{Rev. per min.} = 39 \times \frac{54 \times 112}{18 \times 78} = 168$$

If the number of revolutions of the shaft on which pulley A is mounted is 60, and the number of revolutions required for the shaft on which gear D is mounted, is 110, what diameter pulleys and what size of gears could we employ to transmit the required motion? The speed ratio is $\frac{60}{110} = \frac{6}{11}$. Proceeding as before, we have:

$$\frac{6}{11} = \frac{2 \times 3}{1 \times 11} = \frac{(2 \times 16) \times (3 \times 8)}{(1 \times 16) \times (11 \times 8)} = \frac{32 \times 24}{16 \times 88}$$

The numbers 32 and 24 in the numerator of the last fraction give the diameter of the driven pulley B and the number of teeth of the driven gear D, respectively, and the numbers 16 and 88 in the denominator of the fraction give the diameter of the driving pulley A, and the number of teeth in the driving gear C. In this case, then, pulley A would be 16 inches in diameter, pulley B, 32 inches, gear C would have 88 teeth, and gear D, 24 teeth.

CHAPTER V

CHANGE GEAR CALCULATIONS

WHILE the principles and rules governing the calculation of change gears are very simple, they, of course, presuppose some fundamental knowledge of the use of common fractions. If such knowledge is at hand, the subject of figuring change gears, if once thoroughly understood, can hardly ever be forgotten. It should be impressed upon the minds of all who have found difficulties with this subject that the matter is seldom approached in a logical manner, and is usually grasped by the memory rather than by the intellect.

Lathe Change Gears. When cutting threads in the lathe, the lathe carriage is moved along the bed by means of the lead-screw a certain distance while the work revolves a certain number of times. If the work revolves 12 times while the carriage moves one inch along the bed of the lathe, 12 threads per inch will be cut on the work.

Change gears are used for transmitting the motion from the spindle (which revolves the work) to the lead-screw (which causes the carriage to move along the bed). The number of times that the spindle will revolve while the carriage moves one inch along the lathe bed is determined by the ratio of the change gears. By employing different ratios of change gearing, therefore, different numbers of threads per inch can be cut.

The change gearing may be either *simple* or *compound.* Simple gearing is shown in the accompanying illustration, Fig. 1. When simple gearing is used it is always necessary to use an idler between the gear on the spindle stud and the gear on the lead-screw. As already explained, this idler has no influence on the ratio of the gearing, and can have any number of teeth. Compound change gearing is shown in Fig. 2.

Finding the Lathe Screw Constant. In order to be able to calculate change gears for the lathe, it is necessary first to find the "lathe screw constant." This constant is always the same

for each particular lathe, but it may be different for lathes of different sizes or makes. The lathe screw constant can be determined as follows:

To find the screw constant of a lathe, place gears with an equal number of teeth on the spindle stud and the lead-screw. Then cut a thread on a piece of work in the lathe. The number of threads per inch that will be cut on the work when gears with equal numbers of teeth are placed as directed is called the "screw constant" of the lathe.

For example, put gears with 48 teeth on the spindle stud and on the lead-screw, and any convenient gear on the intermediate stud. Then cut a thread on a piece between the centers. If the

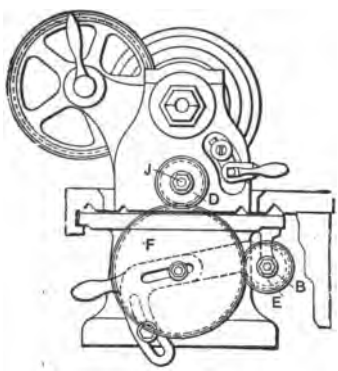


Fig. 1

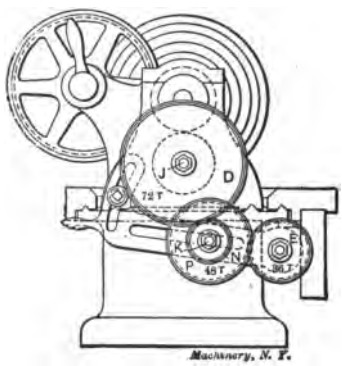


Fig. 2

number of threads per inch when counted as shown on page 18 is found to be 8, the screw constant of this lathe is said to be 8.

To Find the Change Gears for Simple Gearing. When the lathe screw constant has been found, the number of teeth to be used in the change gears for cutting any number of threads within the capacity of the lathe can be determined as follows:

Place the lathe screw constant as the numerator and the number of threads per inch to be cut as the denominator of a fraction and multiply the numerator and denominator by the same number until a new fraction results where the numerator and denominator represent suitable numbers of teeth for the change gears.

In the new fraction the numerator gives the number of teeth in the gear on the spindle stud, and the denominator the number of

teeth in the gear on the lead-screw. This rule can be more easily remembered if written as a formula:

$$\frac{\text{lathe screw constant}}{\text{threads per inch to be cut}} = \frac{\text{teeth in gear on spindle stud}}{\text{teeth in gear on lead-screw}}$$

Assume that 10 threads per inch are to be cut in a lathe where we have found that the lathe screw constant is 6. Also assume that the numbers of teeth in the available change gears of this lathe are 24, 28, 32, 36, 40, etc., increasing by 4 up to 100. By substituting the figures given, in the formula above, and carrying out the calculation, we have:

$$\frac{6}{10} = \frac{6 \times 4}{10 \times 4} = \frac{24}{40}$$

By multiplying both numerator and denominator by 4 we obtain two available gears with 24 and 40 teeth, respectively. The 24-tooth gear goes on the spindle stud, and the 40-tooth gear on the lead-screw. It will be seen that if we had multiplied 6 and 10 by 5, we would have obtained 30 and 50 teeth, which gears are not available in our set of gears with this lathe. Until getting accustomed to figuring of this kind, one can find, by trial only, the correct number by which to multiply the numerator and denominator. The trials must be kept up until both gears have such a number of teeth that they are to be found in the set of change gears accompanying the lathe.

Assume that it is required to cut $11\frac{1}{2}$ threads per inch in the same lathe having the same set of change gears. Then

$$\frac{6}{11\frac{1}{2}} = \frac{6 \times 8}{11\frac{1}{2} \times 8} = \frac{48}{92}$$

It will be found that multiplying by any other number than 8 would not, in this case, give numbers of teeth that could be found in the gears with the lathe. [In order to prevent mistakes, be sure to note that the lathe screw constant differs for different makes and sizes of lathes and must be determined for each particular lathe.]

Compound Gearing. Sometimes it is not possible to obtain gears that will give the required ratio for the thread to be cut in a simple train, and then compound gearing must be employed. The method for finding the number of teeth in the gears in compound gearing is exactly the same as for simple gearing, except that we

divide both numerator and denominator of the fraction giving the ratio of screw constant to threads per inch to be cut, into two factors, and then multiply each "pair" of factors by the same number, in order to obtain the change gears. (One factor in the numerator and one in the denominator make one pair.)

Assume that the lathe screw constant is 6, that the numbers of teeth in the available gears are 30, 35, 40, 45, 50, 55, etc., increasing by 5 up to 100. Assume that it is required to cut 24 threads per inch. We have then,

$$\frac{6}{24} = \text{ratio.}$$

By dividing numerator and denominator of the ratio into two factors and multiplying each pair of factors by the same number, as shown below, we find the gears:

$$\frac{6}{24} = \frac{2 \times 3}{4 \times 6} = \frac{(2 \times 20) \times (3 \times 10)}{(4 \times 20) \times (6 \times 10)} = \frac{40 \times 30}{80 \times 60}$$

The four numbers in the last fraction give the numbers of teeth in the gears which should be used. The gears in the numerator, with 40 and 30 teeth, are the driving gears, and those in the denominator, with 80 and 60 teeth, are the driven gears. Driving gears are, of course, the gear *D*, Fig. 2, on the spindle stud, and the gear *P* on the intermediate stud *K*, meshing with the lead-screw gear. Driven gears are the lead-screw gear, *E*, and the gear *N* on the intermediate stud meshing with the spindle stud gear. Either of the driving gears may be placed on the spindle stud, and either of the driven gears, on the lead-screw.

Assume that $1\frac{3}{4}$ threads per inch are to be cut in a lathe with a screw constant 6, and that the gears available have 24, 28, 32, 36, 40 teeth, etc., increasing by 4 up to 100. Proceeding as before we have:

$$\frac{6}{1\frac{3}{4}} = \frac{2 \times 3}{1 \times 1\frac{3}{4}} = \frac{(2 \times 36) \times (3 \times 16)}{(1 \times 36) \times (1\frac{3}{4} \times 16)} = \frac{72 \times 48}{36 \times 28}$$

This is the case directly illustrated in Fig. 2. The gear with 72 teeth is placed on the spindle stud *J*, the one with 48 on the intermediate stud *K*, meshing with the lead-screw gear. These two gears (72 and 48 teeth) are the *driving* gears. The gears with 36 and 28 teeth are placed on the lead-screw, and on the intermediate stud, as shown, and are the *driven* gears.

The rule for compound change gears, given as a formula, is as follows:

$$\frac{\text{lathe screw constant}}{\text{threads per inch to be cut}} = \frac{\text{product of teeth in driving gears}}{\text{product of teeth in driven gears}}$$

Fractional Threads. Sometimes the lead of a thread is given as a fraction of an inch instead of stating the number of threads per inch. For instance, a thread may be required to be cut, having $\frac{3}{8}$ inch lead. In this case the expression "$\frac{3}{8}$ inch lead" should first be transformed to "number of threads per inch," after which we can proceed to find the change gears as explained on the previous pages. How to find the number of threads per inch when the lead is given is explained in Chapter III. The number of threads (the thread being single) equals:

$$\text{Number of threads per inch} = \frac{1}{\frac{3}{8}} = 1 \div \frac{3}{8} = \frac{8}{3} = 2\frac{2}{3}$$

To find the change gears to cut $2\frac{2}{3}$ threads per inch in a lathe having a screw constant 8 and change gears running from 24 to 100 teeth, increasing by 4, proceed as below:

$$\frac{8}{2\frac{2}{3}} = \frac{2 \times 4}{1 \times 2\frac{2}{3}} = \frac{(2 \times 36) \times (4 \times 24)}{(1 \times 36) \times (2\frac{2}{3} \times 24)} = \frac{72 \times 96}{36 \times 64}$$

Change Gears for Cutting Metric Threads. The metric system of length measurement is in use in practically all countries except in the United States, Great Britain and the British colonies. The unit of length in the metric system is the meter, which equals nearly 39.37 inches (or practically $39\frac{3}{8}$ inches). The subdivisions of the meter are given below:

1 meter = 10 decimeters
1 decimeter = 10 centimeters
1 centimeter = 10 millimeters

In medium and small machine design the unit employed is almost always the millimeter. One millimeter equals 0.03937 inch; one inch equals $\frac{1}{0.03937}$, or 25.4 millimeters, almost exactly.

When screws are made in accordance with the metric system it is not the usual practice to give the number of threads per millimeter or centimeter in the same way as the number of threads per inch is given in the English system. Instead, the lead of the

thread in millimeters is given. A screw thread is said to have 2 millimeters lead, 3 millimeters lead, 4.5 millimeters lead, etc.

It often happens that screws and taps having threads according to the metric system are required. This thread can be cut on a lathe having an English lead-screw, provided change gears with the required number of teeth are used.

The first step in finding the change gears is to find how many threads per inch there are in the screw to be cut, when the lead is given in millimeters. Assume that a screw is required with 3 millimeters lead. How many threads per inch are there in this screw? As there are 25.4 millimeters in one inch, we can find how many threads there would be in one inch, if we find how many times 3 is contained in 25.4; in other words, we divide 25.4 by 3. It is not necessary to carry out the division; simply write it as a fraction in the form $\frac{25.4}{3}$, which implies that 25.4 is to be divided by 3. This fraction now gives the number of threads per inch to be cut. When this fraction has been obtained, proceed as if change gears were to be found for cutting threads with English pitches. Place the lathe screw constant in the numerator of a fraction and the number of threads per inch to be cut in the denominator. If the screw constant of a lathe is 6 and the number of threads to be cut $\frac{25.4}{3}$, as previously found, the ratio of the change gearing is

$$\frac{6}{\frac{25.4}{3}} = \text{ratio.}$$

This seems complicated, but remembering that the line between the numerator and denominator in a fraction means that the numerator is to be divided by the denominator, we get, by carrying out this division:

$$6 \div \frac{25.4}{3} = 6 \times \frac{3}{25.4} = \frac{6 \times 3}{25.4}$$

The fraction $\frac{6 \times 3}{25.4}$ is the ratio of the change gearing required, and all we have to do now is to multiply numerator and denominator of this fraction by the same number until we find suitable numbers of teeth for the change gears. By trial we find that the first whole number by which we can multiply 25.4 so as to get a

whole number as a result, is 5. Multiplying 25.4 by 5 gives us 127. Thus we must have one gear with 127 teeth whenever we cut a metric thread by means of an English lead-screw. The other gear required in this case has 90 teeth, because $5 \times 6 \times 3 = 90$. The calculation would be carried out as shown below:

$$\frac{6 \times 3 \times 5}{25.4 \times 5} = \frac{18 \times 5}{127} = \frac{90}{127}$$

What has just been said can be expressed in the following rule:

To find the change gears for cutting metric pitches with an English lead-screw, place the lathe screw constant multiplied by the number of millimeters lead of the thread to be cut multiplied by 5, in the numerator of the fraction, and 127 as the denominator. The product of the numbers in the numerator gives the number of teeth in the gear on the spindle stud, and 127 is the number of teeth in the gear on the lead-screw.

Written as a formula this rule would be:

$$\frac{\text{lathe screw constant} \times \text{lead of thread to be cut, in millimeters} \times 5}{127} = \frac{\text{teeth in spindle stud gear}}{\text{teeth in lead-screw gear}}$$

As an example, assume that a screw with 2.5 millimeters lead is to be cut on a lathe having a screw constant 8. By placing the given figures in the formula we have:

$$\frac{8 \times 2.5 \times 5}{127} = \frac{100 \ldots . \text{spindle stud gear}}{127 \ldots . \text{lead-screw gear}}$$

Compound Gearing. Sometimes it is necessary to compound the gears because the gear on the spindle stud would have too many teeth, that is, it would be too large to be used in simple gearing. It may also happen that the product of the screw constant $\times$ the lead in millimeters $\times$ 5, is not a whole number, in which case it would be necessary to compound the gears to get whole numbers of teeth.

The method for finding the change gears is exactly the same as the method for compound gears for cutting regular English pitch threads.

Assume that a screw of 6 millimeters lead is to be cut on a lathe with a screw constant 8. By first applying the formula just given, and then dividing the numerator and denominator into factors,

each "pair" of which are multiplied by the same number, we find the change gears as follows:

$$\frac{8 \times 6 \times 5}{127} = \frac{240}{127} = \frac{60 \times 4}{127 \times 1} = \frac{(60 \times 1) \times (4 \times 25)}{(127 \times 1) \times (1 \times 25)}$$
$$= \frac{60 \times 100}{127 \times 25} \begin{matrix} \ldots . \text{driving gears} \\ \ldots . \text{driven gears} \end{matrix}$$

In a case when the lead of the metric screw to be cut is not a whole number but a fraction, it sometimes causes difficulty in dividing up the numerator in two factors that can be multiplied by whole numbers so as to give numbers of teeth for gears which are available. Several trials must often be made.

Assume that the lathe screw constant is 6, and that a screw with 1.25 millimeters lead is to be cut. In this case we would find the change gears as below:

$$\frac{6 \times 1.25 \times 5}{127} = \frac{37.5}{127} = \frac{30 \times 1.25}{127 \times 1} = \frac{(30 \times 1) \times (1.25 \times 40)}{(127 \times 1) \times (1 \times 40)} = \frac{30 \times 50}{127 \times 40}$$

It would not be necessary to write "30 × 1" and "127 × 1" as has been done in this example, but these numbers have been multiplied by 1 simply to preserve a systematic appearance.

In Fig. 3 is shown the arrangement of the gearing when cutting a screw of 1.25 millimeters lead on a lathe with a screw constant 7.

$$\frac{7 \times 1.25 \times 5}{127} = \frac{43.75}{127} = \frac{35 \times 1.25}{127 \times 1} = \frac{(35 \times 1) \times (1.25 \times 100)}{(127 \times 1) \times (1 \times 100)} = \frac{35 \times 125}{127 \times 100}$$

Cutting an English Thread with a Metric Lead-screw. If the lathe has a lead-screw having metric pitch, and it is required to cut a screw with a given number of threads per inch, we must find the "metric screw constant" of the lathe. This is found by placing gears (in simple gearing) with the same number of teeth on the spindle stud and the lead-screw of the lathe, and an idler with any number of teeth, between them, and then cutting a thread on a piece in the lathe. The lead of the thread thus cut, in millimeters, is the metric screw constant of the lathe. Now the method of figuring the change gears when a screw with a given number of threads per inch is to be cut with a lead-screw of metric pitch, is simply the reverse of the method already explained for cutting a metric thread with an English lead-screw.

To find the change gears for cutting English threads with a metric lead-screw, place 127 in the numerator, and the threads

per inch to be cut multiplied by the metric screw constant of the lathe multiplied by 5 in the denominator of the fraction; 127 is the number of teeth in the gear on the spindle stud, and the product of the numbers in the denominator gives the number of teeth in the gear on the lead-screw.

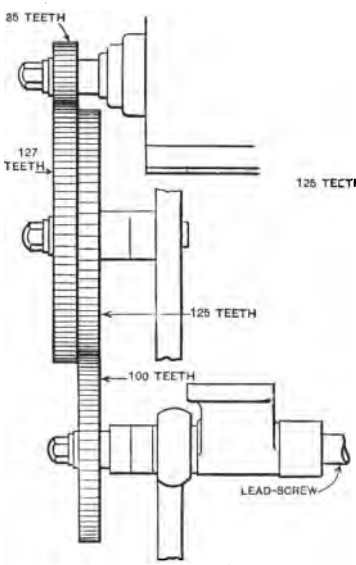


Fig. 3 Cutting a Metric Thread with an English Lead-screw

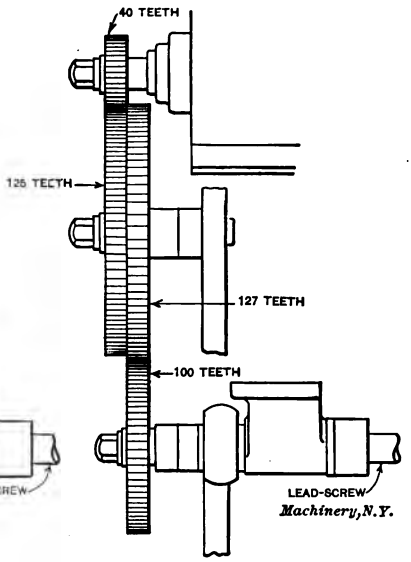


Fig. 4 Cutting an English Thread with a Metric Lead-screw

This rule expressed as a formula would be:

$$\frac{127}{\text{metric screw constant}} \times \frac{\text{threads per inch to be cut}}{} \times 5 = \frac{\text{teeth in gear on spindle stud}}{\text{teeth in gear on lead-screw}}$$

Assume that 5 threads per inch are to be cut in a lathe having a metric screw constant of 4 millimeters. The gears are found directly by using the formula given:

$$\frac{127}{4 \times 5 \times 5} = \frac{127 \ldots \text{spindle stud gear}}{100 \ldots \text{lead screw gear}}$$

It is sometimes necessary to compound the gears in order to obtain gears which are found in the set of change gears provided with the lathe.

Assume that 10 threads per inch are to be cut in a lathe with a metric screw constant of 4 millimeters. To find the gears we would proceed as follows:

$$\frac{127}{4 \times 10 \times 5} = \frac{127}{200} = \frac{127 \times 1}{100 \times 2} = \frac{(127 \times 1) \times (1 \times 40)}{(100 \times 1) \times (2 \times 40)}$$

$$= \frac{127 \times 40}{100 \times 80} \begin{matrix} \ldots . \text{driving gears} \\ \ldots . \text{driven gears} \end{matrix}$$

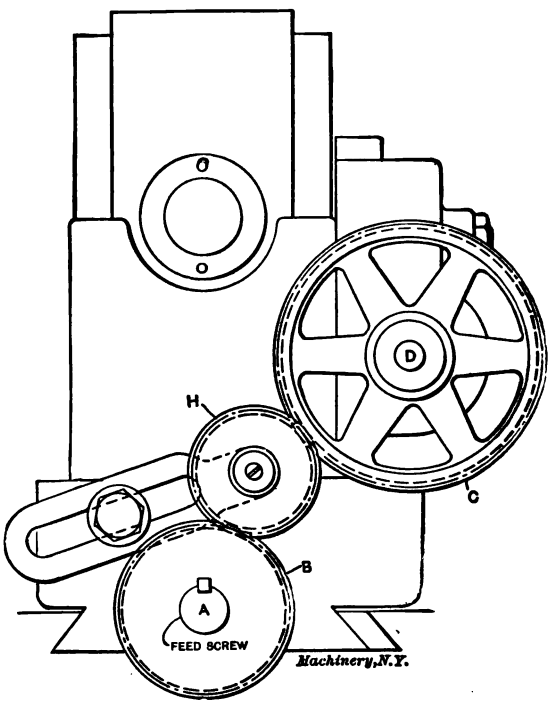


Fig. 5

In Fig. 4 is shown the arrangement of the gearing when cutting a screw, having $12\frac{1}{2}$ threads per inch in a lathe with a metric lead-screw, the metric screw constant being 5 millimeters.

$$\frac{127}{5 \times 12\frac{1}{2} \times 5} = \frac{127}{312.5} = \frac{127 \times 1}{100 \times 3.125}$$

$$= \frac{(127 \times 1) \times (1 \times 40)}{(100 \times 1) \times (3.125 \times 40)} = \frac{127 \times 40}{100 \times 125}$$

Change Gears for Milling Spirals. The method for the figuring of change gears for cutting spirals on the milling machine, is, in principle, exactly the same as that used for figuring change gears for the lathe, but it will be necessary to refer briefly to the construction of the mechanism for connecting the index-head spindle and the feed-screw to make perfectly clear the fundamental ideas governing the selection of change gears.

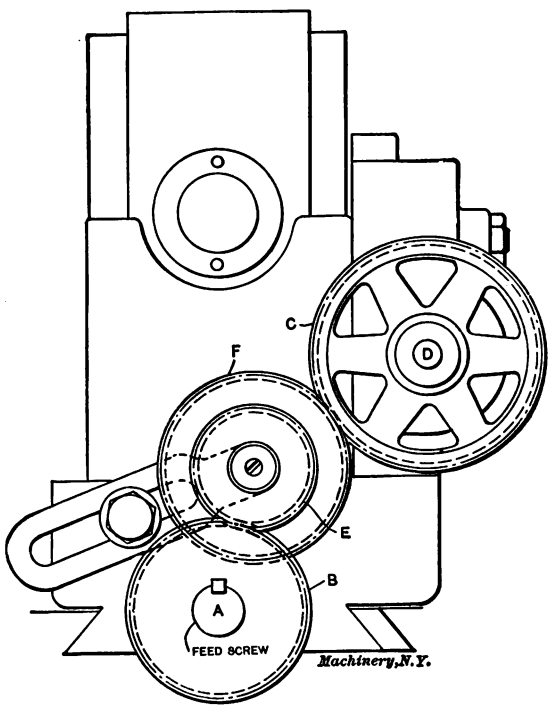


Fig. 6

In Fig. 5 is shown an end view of an index-head for a milling machine, placed on the top of the milling machine table. At *A* is shown the end of the table feed-screw, and *B* is a gear placed on this feed-screw. This gear is commonly called the feed-screw gear, and it imparts motion, through an intermediate gear *H*, to the gear *C* which is placed on the stud *D*; from this stud, in turn, motion is imparted by gearing to the worm of the index-head and from the worm to the worm-wheel mounted on the index-head spindle. Thus, when connected by gearing in this manner, the

index-head spindle may be rotated from the feed-screw. The gear *C* on the stud *D* is called the "worm gear"; this worm gear should not be confused with the worm-wheel which is permanently attached to the index-head spindle.

In Fig. 5 is shown a case of simple gearing, while in Fig. 6 the gears are compounded. In this case *B* still represents the feed-screw gear, while *E* is the gear on the intermediate stud which meshes with *B*, and *F* is the second gear on the same intermediate stud, meshing with gear *C*. The object of the calculation is to find the numbers of teeth in gears *B* and *C* used in a simple train, as in Fig. 5; or in the gears *B*, *E*, *F* and *C* as used in a compound train of gears, as shown in Fig. 6.

The Lead of a Milling Machine. If gears with an equal number of teeth are placed on the feed-screw *A* and the stud *D* in Fig. 5, then the *lead of the milling machine* is the distance the table will travel while the index spindle makes one complete revolution. This distance is a constant used in figuring the change gears, and may vary for different milling machines.

The lead of a helix or spiral is the distance, measured along the axis of the work, in which the spiral makes one full turn around the work. The lead of the milling machine may, therefore, also be expressed as the lead of the spiral that will be cut when gears with an equal number of teeth are placed on studs *A* and *D*, and an idler of suitable size interposed between the gears.

To find the lead of a milling machine, place equal gears on stud *D*, and on feed-screw *A*, Fig. 5, and multiply the number of revolutions made by the feed-screw to produce one revolution of the index-head spindle, by the lead of the thread on the feed-screw.

We can express the rule given as a formula:

$$\text{Lead of milling machine} = \begin{matrix}\text{rev. of feed-screw for} \\ \text{one rev. of index spindle} \\ \text{with equal gears}\end{matrix} \times \begin{matrix}\text{lead of} \\ \text{feed-screw}\end{matrix}$$

Assume that it is necessary to make 40 revolutions of the feed-screw to turn the index-head spindle one complete revolution, when the gears *B* and *C*, Fig. 5, are equal, and that the lead of the thread on the feed-screw of the milling machine is $\frac{1}{4}$ inch; then the lead of the machine equals:

$$40 \times \tfrac{1}{4} \text{ inch} = 10 \text{ inches}$$

Change Gears for Cutting Spirals. As has already been stated the lead of the machine means the distance which the table of the milling machine moves forward in order to turn the work placed on the index-head spindle one complete revolution when change gears with an equal number of teeth are used. If then, for instance, a spiral is to be cut, the lead of which is twice as long as the lead of the machine, change gears of such a ratio must be used that the index-head spindle will turn only one-half a revolution while the table moves forward a distance equal to the lead of the machine.

Assume that we want to cut a spiral having a lead of 20 inches, that is, making one complete turn in a length of 20 inches, and that the lead of the milling machine is 10 inches. Then the ratio between the speeds of the feed-screw and of stud *D* must be 2 to 1, which means that the feed-screw, which is required to turn twice while stud *D* turns once, must have a gear that has only one-half the number of teeth of the gear placed on stud *D*. (See Chapter IV.) If the lead of the machine is 10 inches and the lead of the spiral required to be cut on a piece of work is 30 inches, then the ratio between the speed of the gears would be 3 to 1, which is the same as the ratio between the lead of the spiral to be cut to the lead of the machine. (30 to 10 = 3 to 1, or as it is commonly written 30 : 10 = 3 : 1.)

The rule for finding the change gears can be expressed by a simple formula:

$$\frac{\text{Lead of spiral to be cut}}{\text{Lead of milling machine}} = \frac{\text{number of teeth in gear on worm stud } (D, \text{ Fig. 5})}{\text{number of teeth in gear on feed-screw}}$$

Expressed in words this formula would read:

To find the change gears to be used in a simple train of gearing when cutting spirals in the milling machine, place the lead of the spiral as the numerator and the lead of the milling machine as the denominator of a fraction, and multiply the numerator and denominator by the same number until a new fraction is obtained in which the numerator and denominator give suitable numbers of teeth for the gears.

As an example of this rule, take the case of a milling machine in which there are four threads per inch on the feed-screw and where

40 revolutions of the feed-screw are necessary to make the index spindle turn one complete revolution when gears *B* and *C*, Fig. 5, are equal. It is required on this machine to cut a spiral, the lead of which is 12 inches.

We first find the lead of the machine. As the feed-screw is single and has 4 threads per inch, the lead of the screw thread is $\frac{1}{4}$ inch (see Chapter III), which multiplied by 40 is

$$40 \times \tfrac{1}{4} \text{ inch} = 10 \text{ inches} = \text{lead of machine.}$$

To find the change gears, place the lead of the required spiral as the numerator of a fraction and the lead of the machine as the denominator, and multiply both numerator and denominator by the same number until a new fraction is obtained in which the numerator and denominator express suitable numbers of teeth. Following this rule, then,

$$\frac{12}{10} = \frac{12 \times 4}{10 \times 4} = \frac{48}{40}$$

The gear with 48 teeth is placed on stud *D*, which is required to revolve more slowly than the lead-screw, in order to cut a spiral which is 12 inches, when the spiral cut with equal gears is only 10 inches. The gear having 40 teeth is placed on the feed-screw. An intermediate gear is put between the gear on the feed-screw and the gear on stud *D*; the number of teeth in this intermediate gear has no influence on the speed ratio of feed-screw *A* and stud *D*, but simply serves to transmit motion from one gear to the other. (See Chapter IV.)

Gears for Compound Gearing. If it is not possible to find a set of two gears that will transmit the required motion, it is necessary to use compound gearing. In this case the manner in which the gears are found is exactly the same as the method used for compound gearing in the lathe. The lead of the required spiral is placed as the numerator and the lead of the milling machine as the denominator of a fraction. Then the numerator and denominator are divided into two factors each, and then each "pair" of factors (one factor in the numerator and one in the denominator making one "pair") is multiplied by the same number until suitable numbers of teeth for the change gears are obtained.

As an example, assume that the lead of a machine is 10 inches, and that a spiral having a 48-inch lead is to be cut. Following the method explained we then have

$$\frac{48}{10} = \frac{6 \times 8}{2 \times 5} = \frac{(6 \times 12) \times (8 \times 8)}{(2 \times 12) \times (5 \times 8)} = \frac{72 \times 64}{24 \times 40}$$

The gear having 72 teeth is placed on the stud *D* and meshes with the 24-tooth gear *F* (see Fig. 6) on the intermediate stud. On the same intermediate stud is then placed the gear having 64 teeth, which is driven by the gear having 40 teeth placed on the feed-screw. This makes the gears having 72 and 64 teeth the driven gears, and the gears having 24 and 40 teeth the driving gears, the whole train of gears being driven from the feed-screw of the table.

In general, for compound gearing, the following formula may be used:

$$\frac{\text{Lead of spiral to be cut}}{\text{Lead of machine}} = \frac{\text{product of driven gears}}{\text{product of driving gears}}$$

CHAPTER VI

MILLING MACHINE INDEXING

THE figuring of indexing movements for the dividing head of the milling machine is a subject which many mechanics think complicated, although it is really very simple. Assume that a bolt having a round head as shown in Fig. 1 is required to be milled so that the head becomes hexagonal, that is, so that it has six equal sides, as shown in Fig. 2. The index-head is used for holding the work and for turning or indexing it the required amount for milling each of the six flat surfaces in turn. The index-head is constructed with a worm and worm-wheel mechanism, the worm being on the crank turned when indexing, and the worm-wheel being mounted on the index-spindle to which the work is attached. By moving the crank with its index-pin a certain number of holes in one of the index circles, a certain angular movement can be imparted to the work. The calculating of indexing movements for the milling machine consists in finding how much the index crank requires to be turned in order to produce the required movement for indexing the work.

Calculating the Indexing Movement. Most of the regularly manufactured index-heads use a single-threaded worm engaging with a worm-wheel having 40 teeth. Thus, when the index-crank is turned around one full revolution, the worm is also revolved one complete turn, and this moves the worm-wheel one tooth, or $\frac{1}{40}$ of its circumference. Therefore, in order to turn the worm-wheel and the index-spindle on which it is mounted one full revolution, it is necessary to turn the index-crank 40 revolutions. If we want to revolve the index-spindle one-half revolution, we would have to turn the index-crank 20 revolutions. If we want to turn the index-spindle only one-fourth of a revolution, we turn the index-crank 10 revolutions.

Suppose that we want to mill the hexagonal head of the bolt shown in Fig. 2. As it requires 40 revolutions of the index-crank to revolve the index-spindle once, it evidently requires only $\frac{1}{6}$ of

that number to turn the index-spindle $\frac{1}{6}$ revolution; this is the amount that the work should be turned around or indexed when one side of the hexagon has been milled, and we are ready to mill the next. Consequently, the index crank should be turned around $\frac{40}{6} = 6\frac{2}{3}$ revolutions for milling a hexagon; that is, we first turn the crank 6 full revolutions and then by means of the index-plate we turn it $\frac{2}{3}$ of a revolution. If we use the circle in the index-plate having 18 holes, $\frac{2}{3}$ of a revolution will mean 12 holes in this circle, as 12 is two-thirds of 18 ($12 = \frac{2}{3} \times 18$).

Assume that a piece of work has eight sides regularly spaced (regular octagon). The indexing for each side is found by dividing 40 by 8. Thus $\frac{40}{8} = 5$, represents the number of revolutions of the index-crank for each side indexed and milled.

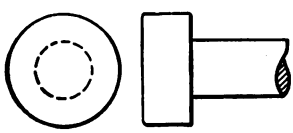


Fig. 1

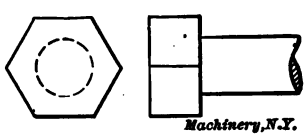


Fig. 2

Assume that it is required to cut nine flutes regularly spaced in a reamer. The index-crank must be turned $\frac{40}{9} = 4\frac{4}{9}$ revolutions in order to index for each flute. The $\frac{4}{9}$ of a revolution would correspond to eight holes in the 18-hole circle, because $\frac{8}{18} = \frac{4}{9}$.

Assume that it is required to cut 85 teeth in a spur gear. The index-crank must be revolved $\frac{40}{85} = \frac{8}{17}$ revolution to index for each tooth. To move the index-crank $\frac{8}{17}$ of a revolution corresponds to moving it 8 holes in the 17-hole circle.

As a general rule, for finding the number of revolutions required for indexing for any regular spacing, with any index-head, the following rule may be used:

To find the number of revolutions of the index-crank for indexing, divide the number of turns required of the index-crank for one revolution of the index-head spindle by the number of divisions required in the work.

[Most standard index-heads are provided with an index-plate fastened directly to the index-spindle for rapid direct indexing. This index-plate is usually provided with 24 holes, so that 2, 3, 4, 6, 8, 12 and 24 divisions can be obtained directly by the use of this direct index-plate without using the regular indexing mechanism. When using this index-plate for rapid direct indexing, no

calculations are required, as the number of divisions obtainable by the use of the different holes in this plate are, as a rule, marked directly at the respective holes.]

Finding the Index-circle to Use. In order to find which index-circle to use and how many holes in that index-circle to move for a certain fractional turn of the index-crank, the numerator and denominator of the fraction expressing the fractional turn are multiplied by the same number until the denominator of the new fraction equals the number of holes in some one index-circle. The number with which to multiply must be found by trial. The numerator of the new fraction then expresses how many holes the crank is to be moved in the circle expressed by the denominator.

Assume that 12 flutes are to be milled in a large tap. Assume that 40 turns of the index-crank are required for one turn of the index-head spindle. First divide 40 by 12 to find the number of turns of the index-crank required for each indexing. Writing out this division as a fraction and carrying out the calculation gives us

$$\frac{40}{12} = 3\frac{4}{12} = 3\frac{1}{3}$$

The fractional turn required is $\frac{1}{3}$ of a revolution. Now multiply, according to the rule given, the numerator and denominator of this fraction by a number so selected that the new denominator equals the number of holes in some one index-circle. Multiplying by 6 would give us

$$\frac{1 \times 6}{3 \times 6} = \frac{6}{18}$$

in which fraction 18 expresses the number of holes in the index-circle to use, and 6 is the number of holes the crank must be moved in this circle to turn the worm-shaft and worm one-third of a revolution.

Most milling machines are commonly furnished with three index-plates, each having six index-circles. The following numbers of holes in the index circles of the three index-plates are commonly used:

15	16	17	18	19	20
21	23	27	29	31	33
37	39	41	43	47	49

Angular Indexing. In Fig. 3 is shown a piece of round stock having two flats milled in such a way that the angle between two lines from the center at right angles to the two surfaces is 35

degrees. In this case the index-head cannot be turned so as to make a certain whole number of moves in one complete revolution of the work, as is done, for instance, when we make four moves in one revolution for milling a square, six moves in one revolution for milling a hexagon, and 80 moves for milling an 80-tooth gear. Instead, we have here given a certain number of degrees which it is required that the work be turned before another cut is taken by the milling cutter.

Indexing for angles is required only when an angle is given which is not such a simple fraction of the whole circle as, for instance, 90 degrees, which is $\frac{1}{4}$ of a complete turn, or 45 degrees, which is $\frac{1}{8}$ of a complete turn, or 60 degrees, which is $\frac{1}{6}$ of a complete turn; the numbers of turns of the index-crank in these cases

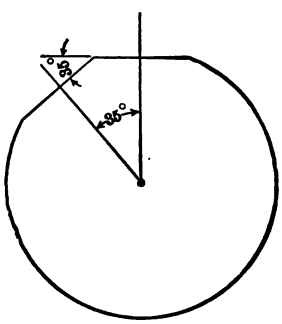


Fig. 3

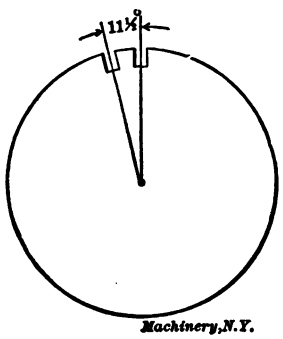


Fig. 4

are determined as explained in the first part of this chapter. But if it be required to index for, say, 19 degrees, the method used is as explained in the following.

Calculating the Movements for Angular Indexing. There are 360 degrees in one complete circle or turn, and assuming that 40 turns of the index-crank are required for one revolution of the work, one turn of the index-crank must equal $\frac{360}{40} = 9$ degrees. Then, when one complete turn of the index-crank equals 9 degrees, two holes in the 18-hole circle, or 3 holes in the 27-hole circle, must correspond to one degree; ($\frac{3}{27} = \frac{2}{18} = \frac{1}{9}$). The first principle or rule for indexing for angles is therefore that two holes in the 18-hole circle or 3 holes in the 27-hole circle equals a movement of one degree of the index-head spindle and the work.

Assume that an indexing movement of 35 degrees is required as shown in Fig. 3. One complete turn of the index-crank equals 9

degrees; we, therefore, first divide the number of degrees for which we wish to index, by 9, in order to find how many complete turns the index-crank should make. The number of degrees left to turn when we have completed the full turns is indexed by taking two holes in the 18-hole circle for each degree. In the case in Fig. 3, $\frac{35}{9} = 3\frac{8}{9}$, which indicates that the index-crank must be turned three revolutions, and then we must index for 8 degrees more or move 16 holes in the 18-hole circle.

Assume that we wish to index $11\frac{1}{2}$ degrees, as shown in Fig. 4. Two holes in the 18-hole circle represent one degree, and consequently one hole represents $\frac{1}{2}$ degree. To index for $11\frac{1}{2}$ degrees we first turn the index-crank one revolution, this being a 9-degree movement. Then to index $2\frac{1}{2}$ degrees we must move the index-crank 5 holes in the 18-hole circle (4 holes for the two whole degrees and one hole for the $\frac{1}{2}$ degree equals the total movement of 5 holes).

Below is shown how this calculation may be carried out to indicate plainly the motion required for this angle:

$11\frac{1}{2}$ deg. = 9 deg. + 2 deg. + $\frac{1}{2}$ deg.
1 turn + 4 holes + 1 hole in the 18-hole circle.

Should it be required to index only $\frac{1}{3}$ degree, this may be made by using the 27-hole circle. In this circle a three-hole movement equals one degree, and a one-hole movement in that circle thus equals $\frac{1}{3}$ degree, or 20 minutes. Assume that it is required to index the work through an angle of 48 degrees 40 minutes. First turn the crank 5 turns for 45 degrees ($5 \times 9 = 45$). Then there are 3 degrees 40 minutes or $3\frac{2}{3}$ degrees left. In the 27-hole circle a three-degree movement corresponds to 9 holes, and a $\frac{2}{3}$-degree movement to 2 holes, making a total movement of 11 holes in the 27-hole circle, to complete the crank movement for 48 degrees 40 minutes. Below is plainly shown how this calculation may be carried out:

48 deg. 40 min. = 45 deg. + 3 deg. + 40 min.
5 turns + 9 holes + 2 holes in the 27-hole circle

Approximate Indexing for Angles. By using the 18- and 27-hole circles, only whole degrees and $\frac{1}{3}$, $\frac{1}{2}$, and $\frac{2}{3}$ of a degree (20, 30, and 40 minutes) can be indexed. Assume, however, that it is required to index for 16 minutes. One whole turn of the index-crank equals 9 degrees or 540 minutes ($9 \times 60 = 540$). To index

for 16 minutes, therefore, requires about $\frac{1}{34}$ of a turn of the index-crank (540 ÷ 16 = 34, nearly). In this case, therefore, we use an index-circle having the nearest number of holes to 34, or the index-circle with 33 holes. A one-hole movement in this circle would approximate the required movement of 16 minutes.

Assume that it is required to index for 55 minutes. We then have 540 ÷ 55 = 10, nearly. In this case there is no index-circle with 10 or approximately 10 holes, but as there is an index-circle with 20 holes, this circle will be used, and the index-crank is moved two holes in that circle instead of one.

Assume that it is required to index for 2 degrees 46 minutes. If we change this to minutes we have 2 degrees = 2 × 60 = 120 minutes, and 46 minutes added to this gives us a total of 166 minutes. Dividing 540 by 166 we have 540 ÷ 166 = 3.253.

Now we multiply this quotient (3.253) by some whole number, so that we obtain a product which equals the number of holes in any one index-circle. The number by which to multiply must be found by trial. In this case we can multiply by 12, obtaining as a product 3.253 × 12 = 39.036. For indexing 2 degrees and 46 minutes we can, therefore, use the 39-hole circle, moving the index-crank 12 holes.

The following is a general rule for *approximate* indexing of angles, for any index-head where 40 revolutions of the index-crank are required for one revolution of the work:

Divide 540 by the total number of minutes to be indexed. If the quotient is approximately equal to the number of holes in any index-circle, the angular movement is obtained by moving one hole in this index-circle. If the quotient does not approximately equal the number of holes in any index-circle, find by trial a number by which the quotient can be multiplied so that the product equals the number of holes in an available index-circle; in this circle, move the index-crank as many holes as indicated by the number by which the quotient has been multiplied.

If the quotient of 540 divided by the total number of minutes is greater than the number of holes in any of the index-circles, the movement cannot be obtained by simple indexing.

CHAPTER VII

THE USE OF FORMULAS

In mathematical and mechanical books and treatises, as well as in articles containing calculations published in engineering journals, formulas are used to a great extent instead of rules expressed in words. In these formulas, signs and symbols are used in order to condense into a small space the essentials of what would otherwise be long and cumbersome rules. The symbols used are generally the letters of the alphabet, and the signs are simply the ordinary signs for arithmetical calculations, with some additional ones necessary for special purposes. Letters from the Greek alphabet are commonly used to designate angles. The Greek letter π (pi) is always used to indicate the proportion of the circumference of a circle to its diameter; π is therefore always equal to 3.1416 in formulas. The most commonly used Greek letters, besides π, are α (alpha), β (beta), and γ (gamma).

Knowledge of algebra is not necessary in order to use successfully formulas for the solving of problems such as occur in ordinary shop practice; but a thorough understanding of the rules and processes of arithmetic is very essential. The symbols or letters used in the formulas simply stand in place of the actual figures or numerical values which are inserted in the formula in each specific case, according to the requirements of the problem to be solved. When these values are inserted, the result required may be obtained by simple arithmetical processes.

There are two main reasons why a formula is preferable to a rule expressed in words: First, the formula is more concise, it occupies less space, and it is possible for the eye to catch at a glance the whole meaning of the rule laid down; secondly, it is easier to remember a short formula than a long rule, and it is, therefore, of greater value and convenience. As it is not always possible to carry a handbook or reference book about, the memory must be relied upon to store up a number of the most frequently occurring mathematical and mechanical rules.

The use of formulas can be explained most readily by actual examples. In the following, therefore, a number of simple formulas will be given, and the values will be inserted so as to show, in detail, the principles involved.

Example 1. When the diameter of a circle is known, the circumference may be found by multiplying the diameter by 3.1416. This rule, expressed as a formula, is:

$$C = D \times 3.1416$$

in which C = circumference of circle, D = diameter of circle.

This formula shows at a glance that no matter what the diameter of the circle be, the circumference is always equal to the diameter times 3.1416. Let it be required to find, for example, the circumference of a circle 24 inches in diameter. If, then, we insert 24 in place of D in the formula, we have:

$$C = 24 \times 3.1416 = 75.3984 \text{ inches}$$

Hence, the formula gives, by means of a simple multiplication, the result required.

Assume that the diameter of a circle is 5.13 inches. The circumference of this circle is found by inserting this value instead of D in the formula:

$$C = 5.13 \times 3.1416 = 16.1164 \text{ inches}$$

Example 2. In spur gears, the outside diameter of the gear can be found by adding 2 to the number of teeth, and dividing the sum obtained by the diametral pitch of the gear. This rule can be expressed very simply by a formula. Assume that we write D for the outside diameter of the gear, N for the number of teeth, and P for the diametral pitch. Then the formula would be:

$$D = \frac{N + 2}{P}$$

This formula reads exactly as the rule given above. It says that the outside diameter D of the gear equals 2 added to the number of teeth N, this sum divided by the pitch P.

If the number of teeth in a gear is 26 and the diametral pitch 4, then simply put these figures in the place of N and P in the formula, and find the outside diameter as in ordinary arithmetic.

$$D = \frac{26 + 2}{4} = \frac{28}{4} = 7$$

D, or the outside diameter, then, is 7 inches.

In another gear the number of teeth is 62 and the pitch 8; find the outside diameter of the gear.

$$D = \frac{62 + 2}{8} = \frac{64}{8} = 8 \text{ inches}$$

From the examples given it will be seen that in formulas, each letter stands for a certain dimension or quantity. When using a formula for solving a problem, replace the letters in the formula by the equivalent figures given in a certain problem, and find the result by means of regular arithmetical calculation.

Example 3. The formula for the horsepower of a steam engine is as follows:

$$\text{H. P.} = \frac{P \times L \times A \times N}{33{,}000}$$

in which H. P. = indicated horsepower of engine;

P = mean effective pressure on piston in pounds per square inch;
L = length of piston stroke in feet;
A = area of piston in square inches;
N = number of strokes of piston per minute.

Assume that $P = 120$, $L = 2$, $A = 320$ and $N = 160$; what would be the horsepower?

If we insert the given values in the formula we have:

$$\text{H. P.} = \frac{120 \times 2 \times 320 \times 160}{33{,}000} = 372.36$$

In formulas the sign for multiplication ($\times$) is often left out between letters, the values of which are to be multiplied. Thus AB means $A \times B$, and the formula

$$\frac{P \times L \times A \times N}{33{,}000} \text{ can also be written } \frac{PLAN}{33{,}000}$$

Thus, if $A = 6$ and $B = 7$, then:

$$AB = A \times B = 6 \times 7 = 42$$

If $A = 9$, $B = 6$ and $C = 7$, then:

$$ABC = A \times B \times C = 9 \times 6 \times 7 = 378$$

It is only the multiplication sign ($\times$) that can be thus left out between the symbols or letters in a formula. All other signs must be indicated the same as in arithmetic.

Parentheses ( ) or brackets [ ] in a formula mean that the expression inside the parentheses or brackets should be considered as one symbol, or in other words, that the calculation inside the parentheses should be carried out by itself, before other calculations are worked.

Examples:

$5 \times (8 + 4) = 5 \times 12 = 60$

$7 \times (18 - 6) + 6\ (4.52 - 1.95) = 7 \times 12 + 6 \times 2.57 = 84 + 15.42 = 99.42$

In the latter example it will be seen that 7 is multiplied by 12, and 6 by 2.57, and then the products of these two multiplications are added. From the order of the numbers $7 \times 12 + 6 \times 2.57$, one might have assumed that the calculation should have been carried out as follows: 7 times 12 = 84, plus 6 = 90, times 2.57 = 231.3. This latter procedure, however, is not correct, as the following rule should be applied:

When several numbers or expressions are connected by the signs +, −, × and ÷, the operations are carried out in the order written, except that *all multiplications should be carried out before the other operations.* The reason for this is that numbers connected by a multiplication sign are only factors of the product thus indicated, which product should be considered by itself as one number. Divisions should be carried out before additions and subtractions, if the division is indicated in the same line with these other processes.

Examples:

$4 \times 7 + 9 - 2 \times 9 = 28 + 9 - 18 = 37 - 18 = 19$

$6 + 7 \times 4 = 6 + 28 = 34$

$72 \div 3 \times 8 = 72 \div 24 = 3$

$8.5 + 16.4 \div 4.1 - 2.5 = 8.5 + 4 - 2.5 = 10$

But $4 \times (7 + 9) - 2 \times 9 = 4 \times 16 - 18 = 64 - 18 = 46$

$(6 + 7) \times 4 = 13 \times 4 = 52$

$(72 \div 3) \times 8 = 24 \times 8 = 192$

$(8.5 + 16.4) \div (4.1 - 2.5) = 24.9 \div 1.6 = 15.56$

In Chapter XIV the meanings of square and cube, and square root and cube root are explained. The squares and square roots as well as the cubes and cube roots of all numbers up to 1,000 (sometimes up to 1,600) are generally given in all standard handbooks.

Example: Find the value of A in the formula

$$A = \sqrt{B^2 + C^2}$$

if $B = 16$ and $C = 12$.

If we insert the given values in the formula, we have

$$A = \sqrt{16^2 + 12^2} = \sqrt{256 + 144} = \sqrt{400} = 20$$

In the same way as we write $2^2 = 2 \times 2$, and $2^3 = 2 \times 2 \times 2$, we can write $2^4 = 2 \times 2 \times 2 \times 2$; and the expression 2^5 would mean that 2 is repeated as a factor five times, or

$$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$$

The expression 2^4 is read "the fourth power of 2" and 6^5, "the fifth power of 6," etc.

In the same way as we may say that the square root means the reverse of square, and the cube root the reverse of cube, so we may

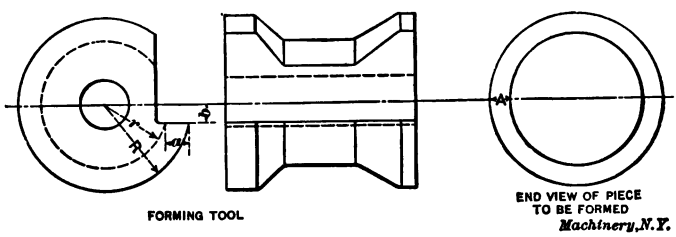


Fig. 1

say that the fourth root is the reverse of the fourth power; that is, if we want the number which repeated as a factor four times gives a given number, we must obtain the fourth root, or $\sqrt[4]{\ }$. Thus $\sqrt[4]{81} = 3$, because $3 \times 3 \times 3 \times 3 = 81$. Similarly we write the fifth root $\sqrt[5]{\ }$; and $\sqrt[5]{32} = 2$, because $2 \times 2 \times 2 \times 2 \times 2 = 32$.

The examples given indicate the principles involved in the use of formulas, and show also how easily formulas may be employed by anyone who has a general understanding of arithmetic. While it would be possible to express in words all the rules required in ordinary shop problems, it is much simpler to make use of formulas. In the following, formulas will be employed wherever required, thus making clear their use in practical work.

A useful application both of the use of formulas and of the square and square root of numbers, is found in the problems occurring when figuring forming tools.

Formulas for Circular Forming Tools. When laying out circular forming tools, such as shown in Fig. 1, the cutting edge, as is well known, must be located a certain amount below the horizontal center line of the tool, in order to provide for sufficient clearance for the cut. On account of this, the actual differences of diameters in the piece of work to be formed cannot be directly copied in the forming tool. The distance A in the piece to be formed must equal the distance a on the forming tool, but as this latter distance is measured in a plane a certain distance b below the horizontal plane through the center of the forming tool, it is evident that the differences of diameters in the tool and the piece to be formed are not the same. A general formula may, however,

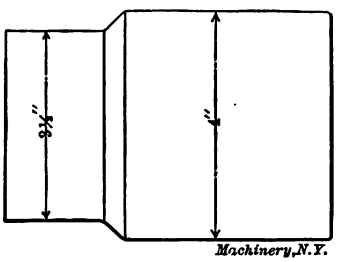


Fig. 2

be deduced, by the use of elementary geometry, by means of which the various diameters of the forming tool may be determined if the largest (or smallest) diameter of the tool, the amount that the cutting edge is below the center, and, of course, the diameters of the piece to be formed, are known.

If R = the largest radius of the tool,

a = difference in radii of steps in the work, and

b = amount cutting edge is below center,

then, if r be the radius required,

$$r = \sqrt{(\sqrt{R^2 - b^2} - a)^2 + b^2}$$

If the smaller radius r is given and the larger radius R sought, the formula takes the form:

$$R = \sqrt{(\sqrt{r^2 - b^2} + a)^2 + b^2}$$

Suppose, for an example, that a tool is to be made to form the piece in Fig. 2. Assume that the largest diameter of the tool is to be 3 inches, and that the cutting edge is to be $\frac{1}{4}$ inch below the

center of the tool. Then the diameter next smaller to 3 inches is found from the formulas given by inserting the given values: $R = 1\frac{1}{2}$ inch, $b = \frac{1}{4}$ inch, and $a = \frac{1}{4}$ inch (half the difference between 4 and $3\frac{1}{2}$ inches; see Fig. 2).
Then

$$r = \sqrt{(\sqrt{(1\tfrac{1}{2})^2 - (\tfrac{1}{4})^2} + \tfrac{1}{4})^2 + (\tfrac{1}{4})^2} = \sqrt{(\sqrt{\tfrac{81}{16}} + \tfrac{1}{4})^2 + \tfrac{1}{16}}$$
$$= \frac{5.017}{4} = 1.254 \text{ inches.}$$

While the formula looks complicated, by means of a table of squares the calculations are easily simplified and can be carried out in three or four minutes. The value of r being 1.254 inch, the diameter to make the smaller step of the forming tool will be 2.508 inches, instead of $2\frac{1}{2}$ inches exact, as would have been the case if the cutting edge had been on the center line.

CHAPTER VIII

HORSEPOWER OF BELTING

THE horsepower which a belt of a given size can transmit depends on the speed with which the belt travels and the working stress advisable to permit in the belt. The speed with which the belt travels, of course, depends on the diameter and number of revolutions per minute of the pulley over which it travels, it being assumed that there is no appreciable slip between the belt and the pulley. If we are to find the horsepower a belt can safely transmit, we must, therefore, consider in our formulas the diameter of the pulley, its number of revolutions per minute, and the permissible working stress in the belt.

Let d = diameter of driving pulley in inches;
v = velocity of belt in feet per minute;
n = number of revolutions of pulley per minute;
S = working stress of belt per inch of width, in pounds;
w = width of belt in inches.

Then:

$$v = \frac{\pi\, dn}{12} = \frac{3.1416\, dn}{12} = 0.2618\, dn$$

$$\text{H. P.} = \frac{Svw}{33{,}000} = \frac{0.2618\, Sdnw}{33{,}000}$$

A commonly used value for the safe working stress per inch of width of single belts is 33 pounds. When this value is adopted, a belt one inch wide, traveling at a rate of 1,000 feet per minute, will transmit one horsepower.

Example. How many horsepower will a single belt $2\frac{1}{2}$ inches wide, traveling over a pulley 12 inches in diameter, transmit, if the pulley makes 200 revolutions per minute? Assume the working stress at 33 pounds per inch of width of belt.

In this example $d = 12$, $n = 200$, $S = 33$ and $w = 2\frac{1}{2}$. If these values are inserted in the horsepower formula given, we have:

$$\text{H. P.} = \frac{0.2618 \times 33 \times 12 \times 200 \times 2.5}{33{,}000} = 1.57$$

A working stress up to 45 pounds per inch of width of belt is permissible for single belts in good condition. If we adopt this latter value for the stress, how many horsepower would the given belt transmit?

We need only to change 33 in the equation just given to 45, and then we have:

$$\text{H. P.} = \frac{0.2618 \times 45 \times 12 \times 200 \times 2.5}{33{,}000} = 2.14$$

If the horsepower to be transmitted is known, the width of belt required may be found by a transposition of the given formula, as follows:

$$w = \frac{\text{H. P.} \times 33{,}000}{Sv} = \frac{\text{H. P.} \times 33{,}000}{0.2618\, Sdn}$$

in which formula the letters denote the same quantities as previously given.

Example: Find the width of single belt required to transmit 20 horsepower with a belt velocity of 1,800 feet per minute.

In this example H. P. = 20, v = 1,800, and S may be assumed to be 45. If we insert these values in the given formula for width of belt, we have:

$$w = \frac{20 \times 33{,}000}{45 \times 1{,}800} = 8.15 \text{ or, say, } 8\tfrac{1}{4} \text{ inches.}$$

In order to reduce the width of a single belt when it becomes too wide, a double belt may be used. The working stress of a double belt per inch of width may be assumed at from 65 to 90 pounds, the latter value being only for belts kept in good condition.

Assume that in the example just given, we use a double belt instead of a single, and assume a working stress of 80 pounds per inch of width of belt. How wide, then, would the belt be?

Substituting 80 for 45, we have:

$$w = \frac{20 \times 33{,}000}{80 \times 1{,}800} = 4.58 \text{ or, say, } 4\tfrac{3}{4} \text{ inches.}$$

As the working stress is an assumed quantity, always somewhat uncertain, it is, of course, not necessary to retain in our formulas so exact a quantity as 0.2618. If this number is given in round figures as 0.25 or $\frac{1}{4}$, we could simplify the given formulas as follows:

$$\text{H. P.} = \frac{Sdnw}{4 \times 33{,}000}$$

$$w = \frac{\text{H. P.} \times 33{,}000 \times 4}{Sdn}$$

As a final example, find the horsepower transmitted by a 5-inch wide double belt, working stress 75 pounds per inch width of belt, if the belt transmits power from a 4-foot pulley running at 200 revolutions per minute.

In this example $w = 5$, $S = 75$, $n = 200$, and $d = 4 \times 12 = 48$ inches. If we insert these values in our simplified formula, we have:

$$\text{H. P.} = \frac{75 \times 48 \times 200 \times 5}{4 \times 33{,}000} = 27.3$$

CHAPTER IX

TIME REQUIRED FOR DRILLING, MILLING AND PLANING

THE feed of a drill in the drill press is the downward motion of the drill per revolution. The feed of a milling cutter is the forward movement of the milling machine table for each revolution of the cutter. Sometimes the feed is expressed as the distance which the drill or the milling machine table moves forward in one minute. In order to avoid confusion, it is, therefore, always best to state plainly in each case whether feed per revolution or feed per minute is meant.

Time Required for Drilling. In order to calculate the time required for drilling a given depth of hole, the number of revolutions per minute of the drill, and the feed per revolution (or the cutting speed, the diameter of the drill and the feed per revolution) must be known.

Assume that a $1\frac{7}{8}$-inch drill makes 80 revolutions per minute and that the feed per revolution is 0.008 inch. How long a time will it require to drill a hole $5\frac{1}{2}$ inches deep? To find the number of revolutions required to drill the full depth of the hole, divide $5\frac{1}{2}$ by 0.008, obtaining the quotient 687.5 or approximately 690 revolutions. As the drill makes 80 revolutions in one minute, we find the total number of minutes required by dividing 690 by 80, the quotient 8.6 being the number of minutes required to drill a hole $5\frac{1}{2}$ inches deep under the given conditions. If, in the foregoing,

T = time required for drilling, in minutes,

L = depth of drilled hole, in inches,

N = number of revolutions per minute of the drill,

F = feed per revolution, in inches,

then

$$T = \frac{L}{N \times F}$$

Expressed as a rule, this formula would be:

To find the time required to drill a hole to a given depth when the feed per revolution of the drill, the depth of the hole, and the

number of revolutions per minute are given, divide the depth of the hole by the number of revolutions per minute multiplied by the feed per revolution.

If the cutting speed of the drill and its diameter are given instead of the number of revolutions, the number of revolutions must first be found before applying the formula given.

If the feed per minute is given, the feed per revolution can be found by dividing the feed per minute by the number of revolutions per minute.

The feed of drills should be about 0.004 inch per revolution for a $\frac{1}{16}$-inch drill, 0.005 inch for a $\frac{1}{4}$-inch drill, 0.008 inch for a $\frac{1}{2}$-inch drill, 0.010 inch for a 1-inch drill, and 0.015 inch for a 2-inch drill. If the drill breaks or chips at the cutting edges, the feed should be reduced.

Time Required for Milling. The time required for milling may be found if the number of revolutions per minute of the cutter, and the feed per revolution (or the cutting speed, the diameter of the cutter and the feed per revolution) are known. If the feed per minute is given, the feed per revolution can be found by dividing the feed per minute by the number of revolutions per minute.

If the length of the cut taken in a milling machine is $8\frac{3}{8}$ inches and the feed is $\frac{1}{64}$ per revolution, how long a time will it take for a cutter making 20 revolutions per minute to traverse the work? As the feed per revolution is $\frac{1}{64}$ inch and the cutter makes 20 revolutions per minute, the feed per minute is $\frac{20}{64}$ or $\frac{5}{16}$ inch. To find the time required for the cutter to traverse the full length of the work, divide the length of the cut, $8\frac{3}{8}$ inches, by the feed in one minute; thus:

$$8\frac{3}{8} \div \frac{5}{16} = \frac{67}{8} \times \frac{16}{5} = \frac{134}{5} = 26\frac{4}{5} = 26.8$$

The time required would thus be 27 minutes, approximately.

If T = time required for the cutter to traverse the work, in minutes,

L = length of cut, in inches,

N = revolutions per minute of the cutter,

F = feed per revolution, in inches,

then

$$T = \frac{L}{N \times F}$$

It will be seen that the form of this formula is the same as that of the formula for the time required for drilling.

If the cutting speed and the diameter of the cutter are given instead of the number of revolutions, the latter number must first be found before the formula above is applied.

The average feed of milling cutters per minute should vary from about 4 inches for a $\frac{1}{2}$-inch mill cutting cast iron, and $1\frac{1}{2}$ inch for the same mill cutting steel, to $1\frac{1}{4}$ inch for a 6-inch cutter on cast iron and $\frac{1}{2}$ inch for the same cutter on steel. Of course, these feeds must be varied with the depth of the cut.

Feed of Planer Tools. The feed of a planer tool is its sidewise motion for each cutting stroke of the table or platen. If for each cutting stroke the tool-carrying head moves $\frac{1}{16}$ inch along the cross-rail, we say that the feed is $\frac{1}{16}$ inch. Each cutting stroke necessitates a return stroke, and in the following, when the expression "number of strokes" is used, it means number of cutting strokes.

Time Required for Planing. The time required for planing a piece of work can be calculated if the feed per stroke, the number of strokes of the planer table per minute, and the width of the work, are known.

Assume that a planer makes 6 strokes per minute, that the feed per stroke is $\frac{3}{32}$ inch, and that the width of the work is 22 inches. Find the time required for planing the work.

As the planer makes 6 strokes per minute and the feed per stroke is $\frac{3}{32}$ inch, the feed per minute is $6 \times \frac{3}{32}$ or $\frac{9}{16}$ inch. The tool must traverse 22 inches to plane the complete work; the traverse in one minute being $\frac{9}{16}$ inch, the total number of minutes required to traverse the work is found by dividing 22 by $\frac{9}{16}$.

$$22 \div \frac{9}{16} = \frac{22}{1} \times \frac{16}{9} = \frac{352}{9} = 39\frac{1}{9} \text{ minutes.}$$

The time required for planing the work is thus 40 minutes, approximately.

This calculation may be summed up in the following formula, applicable to any case where the feed per stroke, the number of strokes per minute, and the width of the work are known:

$$T = \frac{W}{F \times N}$$

In this formula

T = time required for planing, in minutes;
W = width of work, in inches;
F = feed per stroke, in inches;
N = number of strokes per minute.

The formula expressed as a rule would be as follows:

To find the time required for planing when the width of the work, the feed per stroke and the number of strokes per minute, are known, divide the width of the work by the feed times the number of cutting strokes per minute.

CHAPTER X

AREAS OF PLANE FIGURES

THE square has four sides of equal length, and each of the four angles between the sides is a right or 90-degree angle (see Fig. 1). The area of the square equals the length of the side multiplied by itself, or the square of the length of the side. If the side of a square is 14 inches, then the area equals $14 \times 14 = 196$ square inches. If the side is 14 feet, then the area is 196 square feet.

If the area of a square is known, the length of the side equals the square root of the area. Assume that the area of a square equals 1,024 square inches. Then the side equals $\sqrt{1,024} = 32$ inches.

Rectangles. The rectangle, as shown in Fig. 2, has four sides, of which those opposite each other are of equal length, and the four angles between the sides are right or 90-degree angles.

The area of a rectangle is found by multiplying the height or altitude by the length or base. In Fig. 2, B is the altitude and C the base, and the area equals $B \times C$. If $B = 6$ inches, and $C = 11$ inches, then the area equals $6 \times 11 = 66$ square inches.

If the area of a rectangle and the length of its base are known, the height is found by dividing the area by the length of the known base. Either the longer or the shorter side may be considered as the base, the altitude being the side at right angles to the base. If, in Fig. 2, the area of the rect angle is 96 square inches and the side C is 12 inches, then the side $B = 96 \div 12 = 8$ inches.

One square foot equals $12 \times 12 = 144$ square inches. If the area is given in square feet, it can, therefore, be transformed into square inches by multiplying by 144. If the area is given in square inches, it can be transformed into square feet by dividing by 144.

Parallelograms. Two lines are said to be parallel when they have the same direction; when extended, they do not meet or intersect, and the same distance is maintained between the two lines at every point.

Any figure made up of four sides, of which those opposite are parallel, is called a parallelogram. The square and rectangle are parallelograms in which all the angles are right angles. In Fig. 3 is shown a parallelogram where two of the angles are less and two more than 90 degrees. A line drawn from one side of a parallelo-

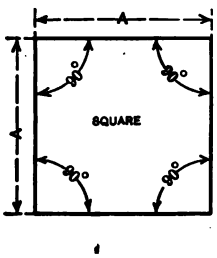


Fig. 1. Square

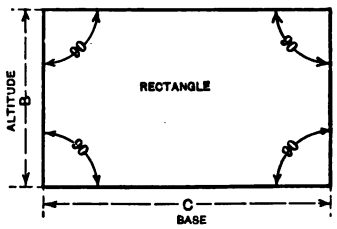


Fig. 2. Rectangle

gram at right angles to the opposite side is called the height or altitude of the parallelogram. In Fig. 3, D is the altitude, and E is the length or base.

The area of a parallelogram equals the altitude multiplied by the base. The area of the parallelogram, in Fig. 3, equals $D \times E$.

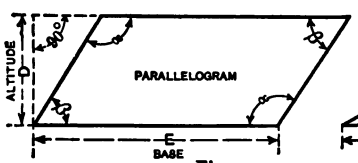


Fig. 3

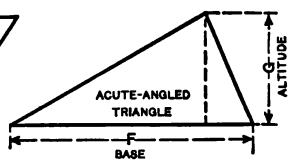


Fig. 4

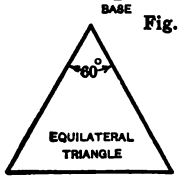


Fig. 5

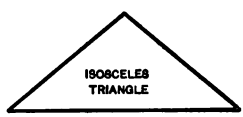


Fig. 6

Figs. 3 to 6. Parallelogram and Triangles

If D is 16 inches, and E, 22 inches, then the area equals $16 \times 22 = 352$ square inches.

If the area and the base are given, the altitude is found by dividing the area by the base.

In parallelograms the angles opposite each other are alike, as indicated in Fig. 3, where the two angles α are equal, and the two angles β also are equal.

Triangles. Any figure bounded by three straight lines is called a triangle. Any one of the three lines may be called the base, and the line drawn from the angle opposite the base at right angles to it is called the height or altitude of the triangle. In Fig. 4, if the side F is taken as the base of the triangle, then G is the altitude.

If all three sides of a triangle are of equal length, as in the one shown in Fig. 5, the triangle is called *equilateral.* Each of the three angles in an equilateral triangle equals 60 degrees.

If two sides are of equal length, as shown in Fig. 6, the triangle is an *isosceles* triangle.

If one angle is a right or 90-degree angle, the triangle is called a *right* or *right-angled* triangle. Such a triangle is shown in Fig. 7; the side opposite the right angle is called the *hypotenuse.*

If all the angles are less than 90 degrees, the triangle is called an *acute* or *acute-angled* triangle, as shown in Fig. 4. If one of the

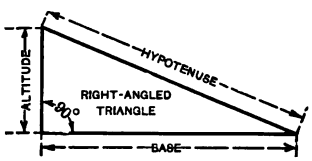


Fig. 7. Right-angled Triangle

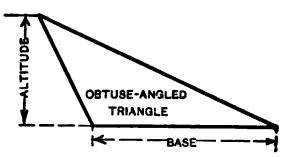


Fig. 8. Obtuse-angled Triangle

angles is larger than 90 degrees, as shown in Fig. 8, the triangle is called an *obtuse* or *obtuse-angled* triangle.

The sum of the three angles in every triangle is 180 degrees. The area of a triangle equals one-half the product of the base and the altitude; thus the area of the triangle shown in Fig. 4 equals $\frac{1}{2} \times F \times G$. If F equals 9 inches, and G, 6 inches, then the area equals $\frac{1}{2} \times 9 \times 6 = 27$ square inches.

If the area and the base of a triangle are known, the altitude can be found by dividing twice the area by the length of the base. If the area and the altitude are known, the base is found by dividing twice the area by the altitude. If the area of a triangle is 180 square inches, and the base is 18 inches, then the altitude equals $(2 \times 180) \div 18 = 20$ inches.

If the length of two sides of a right triangle, Fig. 7, are known, the third side can be found by a simple calculation.

If the base and the altitude are known, the hypotenuse equals the square root of the sum of the squares of the base and the altitude, or: $\text{Hypotenuse} = \sqrt{(\text{base})^2 + (\text{altitude})^2}$

The base and the altitude can be found by similar formulas if the hypotenuse is known.

$$\text{Base} = \sqrt{(\text{hypotenuse})^2 - (\text{altitude})^2}$$

$$\text{Altitude} = \sqrt{(\text{hypotenuse})^2 - (\text{base})^2}$$

Assume that the altitude is 3 feet and the base is 4 feet. Then the hypotenuse $= \sqrt{3^2 + 4^2} = \sqrt{(3 \times 3) + (4 \times 4)} = \sqrt{9 + 16} = \sqrt{25} = 5$ feet.

If the hypotenuse is 10 inches and the altitude 6 inches, then the base equals $\sqrt{10^2 - 6^2} = \sqrt{(10 \times 10) - (6 \times 6)} = \sqrt{100 - 36} = \sqrt{64} = 8$ inches.

Trapezoids. When a figure is bounded by four lines, of which only two are parallel, it is called a trapezoid. The height of a trapezoid is the distance L, Fig. 9, between the two parallel lines H and K. The area of a trapezoid equals one-half the sum of the lengths of the parallel sides multiplied by the height. The area of the trapezoid in Fig. 9 thus equals $\frac{1}{2} \times (H + K) \times L$. If $H = 16$ feet, $K = 24$ feet, and $L = 14$ feet, then the area $= \frac{1}{2}$ $(16 + 24) \times 14 = 280$ square feet.

Trapeziums. When a figure is bounded by four lines, no two of which are parallel, as shown in Fig. 10, it is called a trapezium. The area of a trapezium is found by dividing it into two triangles as indicated by the dash-dotted line in Fig. 10, and finding the area of each of the two triangles, and adding these areas. The dotted lines in Fig. 10 indicate the altitudes of the two triangles into which the trapezium has been divided. If the dimensions of the base and height of the one triangle are R and S, respectively, and of the other T and V, as shown in Fig. 10, then the area of the whole trapezium would be $(\frac{1}{2} \times R \times S) + (\frac{1}{2} \times T \times V)$. Assume that $R = 20$ feet, $S = 17$ feet, $T = 23$ feet, and $V = 9$ feet, then the area of the trapezium $= (\frac{1}{2} \times 20 \times 17) + (\frac{1}{2} \times 23 \times 9) =$ 273.5 square feet.

The Circle. The circle is a plane surface bounded by a curved line called the *periphery* or *circumference*, which is at all points at an equal distance from a point within the circle called the center. The distance from the center of the circle to the periphery is the *radius*, and the distance across the circle through the center is the *diameter*. (See Fig. 11.) It is evident that the radius is one-half

of the diameter. If a line is drawn from one point on the periphery to another point, so that it does not pass through the center, it is called a *chord.*

If the diameter of a circle is known, the circumference is found by multiplying the diameter by 3.1416. Assume that the circumference of a circle is stretched out into a straight line by the circle rolling upon a flat surface and unfolding itself, as shown in Fig. 14, then the length of the straight line would be three times the diameter plus a distance equal to 0.1416 times the diameter; or the whole length of the circumference would be 3.1416 times the diam-

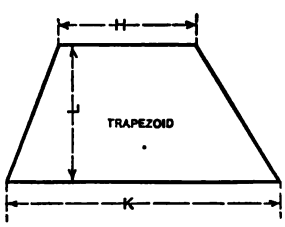


Fig. 9. Trapezoid

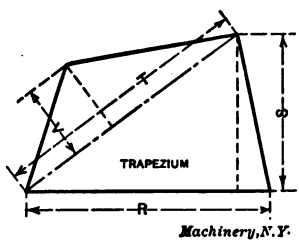


Fig. 10. Trapezium

eter. As the diameter equals 2 × radius, the circumference equals 2 × radius × 3.1416.

If the circumference of a circle is known, the diameter is found by dividing the circumference by 3.1416; the radius is found by dividing the circumference by 2 × 3.1416.

If D = diameter, R = radius, C = circumference, then the previous rules can be written as formulas, thus:

$$D = 2 \times R$$

$$C = 2 \times R \times 3.1416$$

$$C = D \times 3.1416$$

$$R = \frac{C}{2 \times 3.1416} \qquad D = \frac{C}{3.1416}$$

Instead of writing out the number 3.1416, the Greek letter π (pi) is often used; thus, for example, $3\pi = 3 \times 3.1416$.

Example: The diameter of a circle is 6 inches; find its circumference.

Using the formula given, we have:

Circumference = 6 × 3.1416 = 18.8496 inches.

The circumference of a circle is 13.509 inches; find its radius.

$$\text{Radius} = \frac{13.509}{2 \times 3.1416} = 2.150 \text{ inches}$$

The area of a circle equals the square of the radius multiplied by 3.1416; or, the square of the diameter multiplied by 0.7854.

If the area of a circle is known, the radius is found by extracting the square root of the quotient of the area divided by 3.1416.

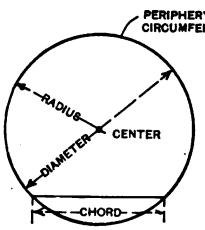


Fig. 11

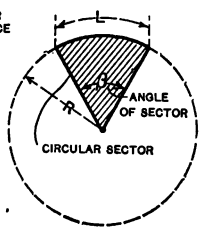


Fig. 12

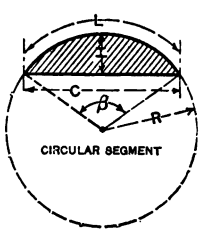


Fig. 13

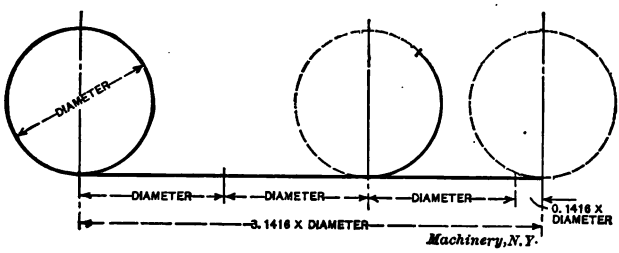


Fig. 14

Figs. 11 to 14. Circles, Sectors and Segments

If D = diameter, R = radius, A = area, then

$$A = R^2 \times 3.1416$$

$$A = \frac{D^2 \times 3.1416}{4} = D^2 \times 0.7854$$

$$R = \sqrt{\frac{A}{3.1416}}$$

Examples: The diameter of a circle is 6 inches, find the area.
Using the formula given, we have:
Area $= 6^2 \times 0.7854 = 6 \times 6 \times 0.7854 = 28.2744$ square inches.
The area of a circle is 95.033 square inches, find the radius.
Using the formula given, we have:
Radius $= \sqrt{95.033 \div 3.1416} = 5.5$ inches.

Circular Sectors. A figure bounded by a part of the circumference of a circle and two radii, as shown in Fig. 12, is called a circular *sector*. The angle β (beta) between the radii is called the angle of the sector, and the length L of the circumference of the circle is called the arc of the sector.

If R = radius of circle of which the sector is a part;
β = angle of sector, in degrees;
L = length of arc of sector;
A = area of sector;
then the formulas below are used:

$$L = \frac{R \times \beta \times 3.1416}{180} = \frac{2 \times A}{R}$$

$$\beta = \frac{180 \times L}{R \times 3.1416}$$

$$A = \frac{L \times R}{2}$$

$$R = \frac{2 \times A}{L} = \frac{180 \times L}{\beta \times 3.1416}$$

If the radius of a circle is $1\frac{1}{2}$ inch, and the angle of a circular sector is 60 degrees, how long is the arc of the sector?

Using the given formula, we have:

$$L = \frac{1\frac{1}{2} \times 60 \times 3.1416}{180} = 1.5708 \text{ inch.}$$

What is the area of the same sector?

From the formula given, we have:

$$A = \frac{1.5708 \times 1\frac{1}{2}}{2} = 1.1781 \text{ square inch.}$$

Circular Segments. A figure bounded by a part of the circumference of a circle and a chord, as shown in Fig. 13, is called a circular *segment*. The distance H from the chord to the highest point of the circular arc is called the height of the segment.

If R = radius, C = length of chord, L = length of arc of segment, H = height of segment, A = area of segment, then the following formulas are used:

$$C = 2 \times \sqrt{H \times (2 \times R - H)}$$

$$R = \frac{C^2 + 4 \times H^2}{8 \times H}$$

$$A = \frac{L \times R - C \times (R - H)}{2}$$

If the angle, β, Fig. 13, is given, instead of the length of arc L, the length of the arc is found by the previously given formula:

$$L = \frac{R \times \beta \times 3.1416}{180}$$

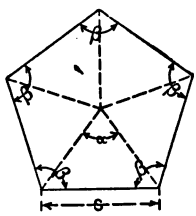


Fig. 15

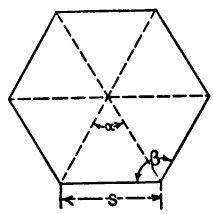


Fig. 16

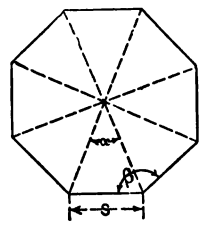


Fig. 17

Regular Polygons

Assume that the radius of a segment is 5 feet and the height 8 inches. How long is the chord of this segment?

First transform 5 feet into inches; $5 \times 12 = 60$ inches. Then apply the formula given:

$C = 2 \times \sqrt{8 \times (2 \times 60 - 8)} = 2 \times \sqrt{896} = 2 \times 29.93 = 59.86$ inches.

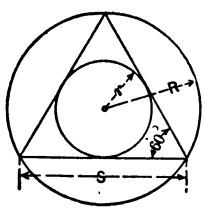


Fig. 18. Equilateral Triangle

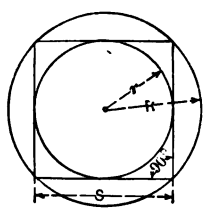
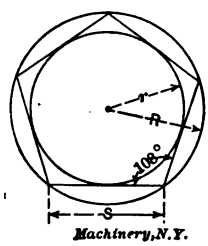


Fig. 19. Square



Machinery, N.Y.

Fig. 20. Regular Pentagon

The length of the chord of a segment is 16 inches and the height 6 inches. How long is the radius of the circle of which the segment is a part?

Applying the formula given, we have:

$$R = \frac{16^2 + 4 \times 6^2}{8 \times 6} = \frac{256 + 144}{48} = 8\tfrac{1}{3} \text{ inches.}$$

Regular Polygons. Any plane surface or figure bounded by straight lines is called a *polygon*. If all the sides are of equal length

and the angles between the sides are equal, the figure is called a *regular* polygon.

A regular polygon having five sides is shown in Fig. 15. The length of each of the five sides equals S, and each of the angles between the sides equals β.

A regular polygon with five sides is called a *pentagon;* one with six sides (Fig. 16), a *hexagon;* one with seven sides (Fig. 22), a *heptagon;* and one with eight sides (Fig. 17), an *octagon.* When a regular polygon has only three sides (Fig. 18), it becomes an equilateral triangle, and when it has four sides (Fig. 19) a square.

A circle may be drawn so that it passes through all the angle-points of a regular polygon, as shown in Figs. 18 to 23 inclusive;

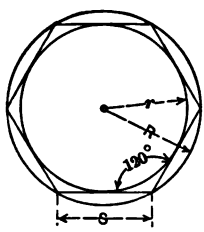


Fig. 21. Regular Hexagon

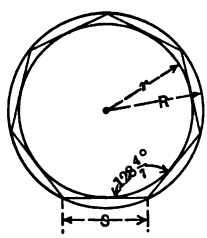


Fig. 22. Regular Heptagon

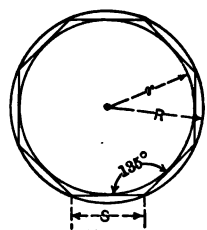


Fig. 23. Regular Octagon

such a circle (with the radius R) is said to be *circumscribed* about the polygon. The smaller circle in the same illustrations (with the radius r) which touches or is tangent to the sides of the polygon, is said to be *inscribed* in the polygon. The centers of the circumscribed and inscribed circles are located at the same point. If the angle-points of the polygon are connected by lines with this center, as shown by the dotted lines in Figs. 15, 16 and 17, the polygon is divided into a number of triangles of equal size and shape. The number of triangles equals the number of sides in the polygon.

The angle α (alpha) of each of these triangles at the center (see Fig. 15) can be determined for any polygon when the number of sides is known. This angle, in degrees, equals 360 divided by the number of sides in the regular polygon, or expressed as a formula, if N equals the number of sides:

$$\alpha = \frac{360}{N}$$

The angle β between two adjacent sides of the polygon (see Fig. 15) equals α subtracted from 180, or:

$$\beta = 180 - \alpha$$

The area of a polygon can be found by dividing it into triangles, as shown in Figs. 15, 16 and 17. After having measured the base and height of one triangle and calculated its area, the area of the whole polygon is found by multiplying the area of one triangle by the number of triangles or sides.

For the more commonly used regular polygons, the formulas in the following give the area directly when the length of the side is known.

Equilateral Triangles. The sum of the three angles in any triangle equals 180 degrees, as already mentioned. Each of the angles in an equilateral triangle, therefore, equals $\frac{1}{3}$ of 180 degrees, or 60 degrees.

The radius r of the circle inscribed in an equilateral triangle equals the side multiplied by 0.289.

The radius R of the circumscribed circle equals the side multiplied by 0.577.

If the radius of the circumscribed circle is known, the side is found by multiplying the radius by 1.732.

If the radius of the inscribed circle is known, the side is found by multiplying the radius by 3.464.

The area of an equilateral triangle equals the square of the side multiplied by 0.433; or, the square of the radius of the circumscribed circle multiplied by 1.299; or, the square of the radius of the inscribed circle multiplied by 5.196.

If r = radius of inscribed circle;
R = radius of circumscribed circle;
S = length of side;
A = area of equilateral triangle;

then the previous rules may be expressed in formulas as follows:

$$r = 0.289 \times S$$
$$R = 0.577 \times S$$
$$S = 1.732 \times R = 3.464 \times r$$
$$A = 0.433 \times S^2 = 1.299 \times R^2 = 5.196 \times r^2$$

The Square. Each of the angles between the sides of a square is a 90-degree or right angle.

The radius of the inscribed circle equals one-half of the side.

The radius of the circumscribed circle equals the side multiplied by 0.707.

The side of a square equals twice the radius of the inscribed circle, or 1.414 times the radius of the circumscribed circle.

The area equals the square of the side. The area also equals the square of the radius of the circumscribed circle multiplied by 2; or, the square of the radius of the inscribed circle multiplied by 4.

Using the same meaning for the letters as before, the previous rules may be expressed in formulas as follows:

$$r = 0.5 \times S$$
$$R = 0.707 \times S$$
$$S = 1.414 \times R = 2 \times r$$
$$A = S^2 = 2 \times R^2 = 4 \times r^2$$

The Pentagon. In the pentagon (Figs. 15 and 20) the angle β between the sides equals 108 degrees. This is found by the formulas previously given as shown below:

$$N = \text{number of sides} = 5$$
$$\alpha = \frac{360}{N} = \frac{360}{5} = 72 \text{ degrees}$$
$$\beta = 180 - \alpha = 180 - 72 = 108 \text{ degrees}$$

The following formulas are used for finding the radii of the circumscribed and inscribed circles, the side and the area of regular pentagons:

$$r = 0.688 \times S$$
$$R = 0.851 \times S$$
$$S = 1.176 \times R = 1.453 \times r$$
$$A = 1.720 \times S^2 = 2.378 \times R^2 = 3.633 \times r^2$$

The Hexagon. In the hexagon (Figs. 16 and 21) the length of the side S equals the radius R of the circumscribed circle so that each of the six triangles formed, when lines are drawn from the center to the angle-points, are equilateral triangles. The angle β between two adjacent sides equals the sum of two angles in two of the equilateral triangles and, consequently, equals $60 + 60 = 120$ degrees.

Using the same letters as previously given in the formulas, we have for the hexagon:

$$r = 0.866 \times S$$
$$R = S$$
$$S = R = 1.155 \times r$$
$$A = 2.598 \times S^2 = 2.598 \times R^2 = 3.464 \times r^2$$

The Heptagon. The heptagon, Fig. 22, has seven sides, and the angle between two adjacent sides is found by the formulas already given, as shown below:

$$N = \text{number of sides} = 7$$
$$\alpha = \frac{360}{N} = \frac{360}{7} = 51\tfrac{3}{7} \text{ degrees}$$
$$\beta = 180 - 51\tfrac{3}{7} = 128\tfrac{4}{7} \text{ degrees}$$

Using the same letters as in the formulas previously given, we have for the heptagon:

$$r = 1.038 \times S$$
$$R = 1.152 \times S$$
$$S = 0.868 \times R = 0.963 \times r$$
$$A = 3.634 \times S^2 = 2.736 \times R^2 = 3.371 \times r^2$$

The Octagon. The angle β between two adjacent sides of the octagon, as shown in Figs. 17 and 23, is 135 degrees.

Using the same meaning for the letters as previously given, the formulas for the octagon are:

$$r = 1.207 \times S$$
$$R = 1.307 \times S$$
$$S = 0.765 \times R = 0.828 \times r$$
$$A = 4.828\, S^2 = 2.828 \times R^2 = 3.314 \times r^2$$

CHAPTER XI

VOLUMES OF SOLIDS

THE cube is a solid body having six surfaces or faces, all of which are squares; as all the faces are squares, all the sides are of equal length (see Fig. 1).

If the side of a face of a cube equals S, the volume equals $S \times S \times S$ or, as it is commonly written, S^3.

Assume that the length of the side of a cube equals 3 inches; then the volume equals $3 \times 3 \times 3 = 27$ cubic inches.

When the volume of a cube is known, the length of the side is found by extracting the cube root of the volume.

Assume that the volume of a cube equals 343 cubic inches. If we extract the cube root of 343, we find that the side of the cube is 7 inches.

One cubic foot equals $12 \times 12 \times 12 = 1728$ cubic inches; therefore, a volume given in cubic feet can be transformed into cubic inches by multiplying by 1728; if the volume is given in cubic inches it can be transformed into cubic feet by dividing by 1728.

Volume of Prisms. A solid body, the sides of which are all rectangles, and the ends of which are either rectangles or squares is commonly called a *square prism.* Opposite surfaces or faces are parallel, and all the angles are right angles. A square prism is shown in Fig. 2, where L is its length, W its width, and H its height. The volume of a square prism equals the length times the width times the height, or, expressed as a formula, if V = volume,

$$V = L \times W \times H$$

Assume that $L = 20$ inches, $W = 4$ inches, and $H = 5$ inches, then volume $= 20 \times 4 \times 5 = 400$ cubic inches.

A solid body having the end faces parallel, and the lines along which the other faces intersect or meet parallel, is called a *prism.* The two parallel end faces are called *bases.* The length, height, or altitude L, Fig. 3, of a prism is the distance between the bases, measured at right angles to the base surfaces.

The volume of a prism equals the area of the base multiplied by the length or height of the prism. The area of the base must, therefore, first be found before the volume can be obtained. If the base is a triangle, parallelogram, trapezoid, trapezium or a regular polygon, its area is found by the rules given in Chapter X. If it is a polygon which is not regular, it can always be divided into

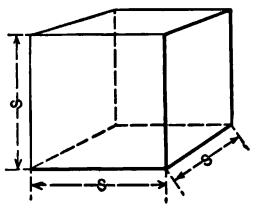


Fig. 1. Cube

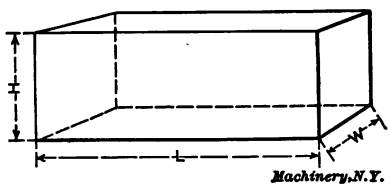


Fig. 2. Square Prism

triangles, and the area of each of the triangles can be calculated, and these areas added together to obtain the area of the whole polygon.

Assume that it is required to find the volume of a prism, the base of which is a regular hexagon having a side S; the length of the prism is L. The volume of this prism is

$$2.598 \times S^2 \times L$$

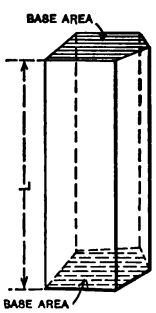


Fig. 3. Prism

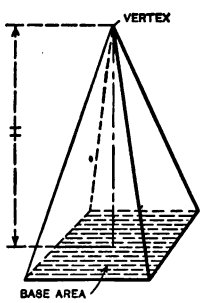


Fig. 4. Pyramid

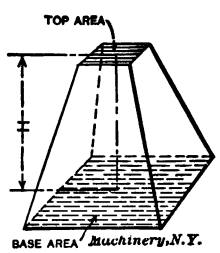


Fig. 5. Frustum of Pyramid

[See Chapter X for formula for area of hexagon.]

If, in this example, S equals $1\frac{1}{2}$ inch, and L equals 9 inches, then the volume equals

$$2.598 \times 1\tfrac{1}{2}^2 \times 9 = 2.598 \times 1.5 \times 1.5 \times 9 = 52.6095 \text{ cubic inches}$$

Volume of a Pyramid. A solid body having a polygon for the base and a number of triangles all having a common vertex for

the sides is called a *pyramid.* In Fig. 4 a pyramid is shown where the base has four sides and the side surfaces are made up of triangles having two equal sides. If a line is drawn from the vertex of the pyramid at right angles to the base, the length of this line is the altitude or height H of the pyramid.

The volume of a pyramid equals the base area multiplied by one-third of the height. It is, therefore, necessary to find the base area before the volume can be found.

Assume that it is required to find the volume of a pyramid, the base of which is a regular pentagon, having a side S; the height of the pyramid is H. The volume of the pyramid equals

$$1.720 \times S^2 \times \tfrac{1}{3} \times H \text{ (area of base } \times \text{ one-third the height).}$$

[See Chapter X for formula for area of pentagon.]

If $S = 2$ inches and $H = 9$ inches, then the volume equals

$$1.720 \times 2^2 \times \tfrac{1}{3} \times 9 = 1.720 \times 2 \times 2 \times 3 = 20.640 \text{ cubic inches}$$

A *frustum of a pyramid* is shown in Fig. 5. It is a pyramid from which the top has been cut off, the top surface being parallel to the base. The height of a frustum of a pyramid is the length of a line drawn from the top surface at right angles to the base.

The volume of a frustum of a pyramid can be found when the height, the top area, and the base area are known.

If V = volume of frustum of a pyramid;
H = height of frustum;
A_1 = area of top;
A_2 = area of base;

then

$$V = \frac{H}{3} \times (A_1 + A_2 + \sqrt{A_1 \times A_2})$$

Assume, for example, that the base of a frustum of a pyramid is a square, and that the side of the square is 5 inches. The top area is, of course, also a square; assume the side of this to be 2 inches. The height of the frustum is 6 inches. By first calculating the base and top areas and then inserting the values in the formula given, the volume is obtained.

Volume $= \frac{6}{3} \times (5^2 + 2^2 + \sqrt{5^2 \times 2^2}) = 2 \times (25 + 4 + \sqrt{25 \times 4})$
$= 2 \times (25 + 4 + 10) = 78$

The Prismoidal Formula. The prismoidal formula is a general formula by which the volume of any prism pyramid or frustum

of a pyramid, and the volume of any solid body bounded by regular curved surfaces may be found.

If A_1 = area at one end of the body;
A_2 = area at other end;
A_m = area of a middle section between the two end surfaces;
H = height of the body;
V = volume of body;
then

$$V = \frac{H}{6} \times (A_1 + 4\,A_m + A_2)$$

As this formula applies to all regular solid bodies, it is useful to remember. For ordinary calculations, however, the formulas given on the two previous pages, for each kind of solid, should be used because of greater simplicity.

Volume of a Cylinder. A solid body, as shown in Fig. 6, having circular and parallel end faces of equal size, is called a *cylinder*. The two parallel faces are called *bases*. The height or altitude H of a cylinder is the distance between the bases measured at right angles to the base surfaces.

The volume of a cylinder equals the area of the base multiplied by the height. The area of the base, must, therefore, first be found before the volume can be obtained. If the diameter of the base is D, the area of the base equals $0.7854\,D^2$. The volume of the cylinder then equals:

$$0.7854 \times D^2 \times H$$

If $D = 3$ inches and $H = 5$ inches, then the volume equals:

$$0.7854 \times 3^2 \times 5 = 0.7854 \times 3 \times 3 \times 5 = 35.343 \text{ cubic inches.}$$

Volume of a Cone. A solid body having a circular base and the sides inclined so that they meet at a common vertex, the same as in a pyramid, is called a cone. (See Fig. 7.) If a line is drawn from the vertex of the cone at right angles to the base, the length of this line is the altitude or height H of the cone.

The volume of a cone equals the base area multiplied by one-third of the height. It is, therefore, necessary to find the area of the base circle before the volume can be found. If the diameter of the base area equals D, then the area equals $0.7854\,D^2$, and this multiplied by one-third of the height H gives us the volume:

$$0.7854 \times D^2 \times \tfrac{1}{3} \times H = \tfrac{1}{3} \times 0.7854 \times D^2 \times H = 0.2618 \times D^2 \times H$$

If the diameter of the base of a cone equals 4 inches and the height 6 inches, then the volume equals:

$$0.2618 \times 4^2 \times 6 = 0.2618 \times 4 \times 4 \times 6 = 25.1328 \text{ cubic inches}$$

A frustum of a cone is shown in Fig. 8. It is a cone from which the top has been cut off, the top surface being a circle parallel to the base. The height H of a frustum of a cone is the length of a line drawn from the top surface at right angles to the base.

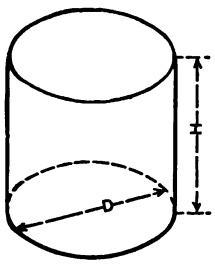


Fig. 6. Cylinder

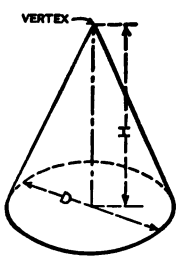


Fig. 7. Cone

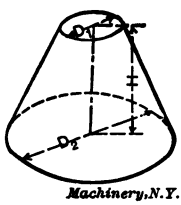


Fig. 8. Frustum of Cone

The volume of a frustum of a cone can be found when the diameters of the top and base circles, and the height are known.

If V = volume of frustum of a cone;
H = height of frustum;
D_1 = diameter of top circle;
D_2 = diameter of base circle;

then

$$V = 0.2618 \times H \times (D_1^2 + D_2^2 + [D_1 \times D_2])$$

Assume, for example, that the diameter of the base of a frustum of a cone is 5 inches, and that the diameter of the top circle is 2 inches. The height of the frustum is 6 inches. By inserting these values in the formula given we have:

$$V = 0.2618 \times 6 \times (2^2 + 5^2 + [2 \times 5]) = 0.2618 \times 6 \times (4 + 25 + 10)$$
$$= 0.2618 \times 6 \times 39 = 61.2612 \text{ cubic inches}$$

Volume of a Sphere, Spherical Sector, Segment and Zone. The name *sphere* is applied to a solid body shaped like a ball or globe, that is, bounded by a surface which at all points is at the same distance from a point inside of the sphere called its center. The diameter of a sphere is the length of a line drawn from a point on the surface through the center to the opposite side.

The volume of a sphere equals 3.1416 multiplied by four-thirds of the cube of the radius, or 3.1416 multiplied by one-sixth of the cube of the diameter.

If R = radius of the sphere, D = diameter, and V = volume, this rule can be written in the form of formulas thus:

$$V = 3.1416 \times \tfrac{4}{8} \times R^3 = 4.1888 \times R^3$$
$$V = 3.1416 \times \tfrac{1}{6} \times D^3 = 0.5236 \times D^3$$

If the volume of a sphere is known, the radius can be found by extracting the cube root of the quotient of the volume divided by

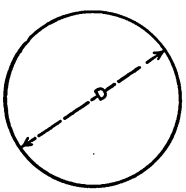


Fig. 9. Sphere

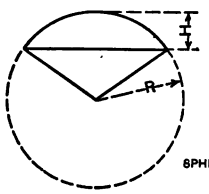


Fig. 10. Spherical Sector

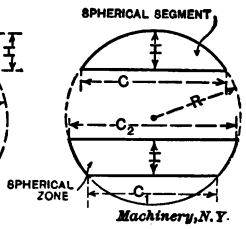


Fig. 11. Spherical Segment and Zone

4.1888; the diameter can be found by extracting the cube root of the quotient of the volume divided by 0.5236.

Written as formulas, these rules are:

$$R = \sqrt[3]{\frac{V}{4.1888}} \qquad D = \sqrt[3]{\frac{V}{0.5236}}$$

A *spherical sector* is a part of a sphere bounded by a section of the spherical surface and a cone, having its vertex at the center of the sphere, as shown in Fig. 10. The volume of a spherical sector can be found if the radius R and the height H, Fig. 10, are known.

The formula for the volume V is

$$V = 2.0944 \times R^2 \times H$$

Assume that the length of the radius of a spherical sector is 15 inches and the height is 4 inches. Then the volume equals

$2.0944 \times 15^2 \times 4 = 2.0944 \times 15 \times 15 \times 4 = 1884.96$ cubic inches.

A *spherical segment* is a part of a sphere bounded by a portion of the spherical surface and a plane circular base, as shown in Fig. 11.

The volume of a spherical segment can be found when the radius of the sphere and the height H of the segment, or the diameter C of the base of the segment and its height H, are known.

If V = volume of segment;
H = height of segment;
R = radius of the sphere of which the segment is a part;
C = diameter of the base of the segment;
then,

$$V = 3.1416 \times H^2 \times \left(R - \frac{H}{3}\right)$$

$$V = 3.1416 \times H \times \left(\frac{C^2}{8} + \frac{H^2}{6}\right)$$

Assume that the height of a spherical segment is 6 inches and the radius 8 inches, then the volume is

$$3.1416 \times 6^2 \times (8 - 6 \div 3) = 3.1416 \times 6 \times 6 \times (8 - 2) =$$
$$3.1416 \times 6 \times 6 \times 6 = 678.5856 \text{ cubic inches.}$$

A *spherical zone* is bounded by a part of a spherical surface, and by two parallel circular bases, as shown in Fig. 11, where C_1 and C_2 are the diameters of the circular bases of the zone, and H its height.

The volume of a spherical zone can be found when the height of the segment and the two base diameters are known

If V = volume of zone;
C_1 = diameter of the smaller base circle;
C_2 = diameter of the larger base circle;
H = height of zone;
then

$$V = 0.5236 \times H \times \left(\frac{3\,C_1^2}{4} + \frac{3\,C_2^2}{4} + H^2\right)$$

Assume that the diameter $C_1 = 3$ inches, the diameter $C_2 = 4$ inches, and the height of the segment equals 1 inch, then the volume is

$$0.5236 \times 1 \times \left(\frac{3 \times 3^2}{4} + \frac{3 \times 4^2}{4} + 1^2\right)$$

$$= 0.5236 \times 1 \times \left(\frac{27}{4} + \frac{48}{4} + 1\right) = 0.5236 \times 1 \times 19.75$$

$$= 10.3411 \text{ cubic inches.}$$

[If a plane parallel with the end faces and passing through the center of the sphere intersects the zone, consider the zone as two zones, one zone being on each side of the center. Calculate the volume of each, and add these to find the total volume.]

CHAPTER XII

SPECIFIC GRAVITY AND WEIGHTS OF BAR STOCK AND CASTINGS

THE expression "specific gravity" indicates how many times a certain volume of a material is heavier than an equal volume of water. If it is found, for example, that one cubic inch of steel weighs 7.8 times as much as one cubic inch of pure water, the specific gravity of steel is 7.8.

As the density of water differs slightly at different temperatures, it is usual to make comparisons on the basis that the water has a temperature of 62 degrees F. The weight of one cubic inch of pure water at 62 degrees F. is 0.0361 pound. If the specific gravity of any material is known, the weight of a cubic inch of the material can, therefore, be found by multiplying its specific gravity by 0.0361.

The specific gravity of cast iron, for example, is 7.2. The weight of one cubic inch of cast iron is found by multiplying 7.2 by 0.0361. The product, 0.260, is the weight of one cubic inch of cast iron.

As there are $12 \times 12 \times 12 = 1{,}728$ cubic inches in one cubic foot, the weight of a cubic foot is found by multiplying the weight of a cubic inch by 1,728.

If the weight of a cubic inch of a material is known, the specific gravity is found by dividing the weight per cubic inch by 0.0361.

The weight of a cubic inch of gold is 0.697 pound. The specific gravity of gold is then found by dividing 0.697 by 0.0361. The quotient, 19.32, is the specific gravity of gold.

If the weight per cubic inch of any material is known, the weight of any volume of the material is found by multiplying the weight per cubic inch by the volume expressed in cubic inches. If brass weighs 0.289 pound per cubic inch, 16 cubic inches of brass, of course, weigh $0.289 \times 16 = 4.624$ pounds. In an example of this kind, if the specific gravity is known, instead of the weight per cubic inch, this latter weight is first found by the rule previously given for finding the weight per cubic inch from the specific gravity.

If the specific gravity of tool steel is 7.85, what is the weight of 12 cubic inches of tool steel? The weight of one cubic inch is found by multiplying 7.85 by 0.0361. The product, 0.283, is then multiplied by 12 to find the weight of 12 cubic inches: $0.283 \times 12 = 3.396$ pounds.

Weight of Bar Stock. The weight of a piece of round bar stock, as shown in Fig. 1, can be found by first calculating the volume of the piece. When the volume is found in cubic inches,

TABLE OF SPECIFIC GRAVITY AND WEIGHT PER CUBIC INCH OF VARIOUS METALS AND ALLOYS

Metal	Specific Gravity	Weight in Pounds per Cubic Inch
Aluminum	2.56	0.092
Antimony	6.71	0.242
Bismuth	9.80	0.354
Brass	8.00	0.289
Copper	8.82	0.318
Gold	19.32	0.697
Iron, cast	7.20	0.260
" pure	7.77	0.280
" wrought	7.70	0.278
Lead	11.37	0.410
Manganese	8.00	0.289
Mercury	13.58	0.490
Nickel	8.80	0.318
Platinum	21.50	0.776
Silver	10.50	0.379
Steel, machine and tool	7.85	0.283
Tin	7.29	0.263
Tungsten	17.60	0.635
Vanadium	5.50	0.199
Zinc	7.15	0.258

the weight is found by multiplying the volume by the weight of the material per cubic inch, as already explained.

If the diameter D, Fig. 1, of a piece of round tool steel bar is 2 inches, and the length L is 7 inches, the volume of this piece equals $0.7854 \times$ square of diameter $\times$ length, or $0.7854 \times 2^2 \times 7 = 0.7854 \times 2 \times 2 \times 7 = 21.991$ cubic inches. The volume in cubic inches having been found, it is multiplied by the weight of tool steel per cubic inch, which is 0.283 pound, as given in the accompanying table of specific gravity and weight per cubic inch of various metals and alloys. The weight of the bar is then $21.991 \times 0.283 = 6.2235$ pounds.

If the specific gravity is given instead of the weight per cubic inch, find the weight per cubic inch, as previously explained.

The weight of a square bar, as shown in Fig. 2, can be calculated when the width across flats, W, the length of the bar, L, and the weight of one cubic inch of the material from which the bar is made, are known.

Assume that the width across flats is $2\frac{1}{2}$ inches, that the length is 11 inches, and that the bar is made from brass; the volume of this bar equals the area of its end section multiplied by its length, or, in this case, $2\frac{1}{2} \times 2\frac{1}{2} \times 11 = 68\frac{3}{4}$ cubic inches. The weight of

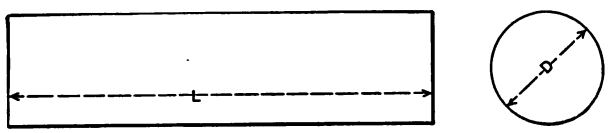


Fig. 1

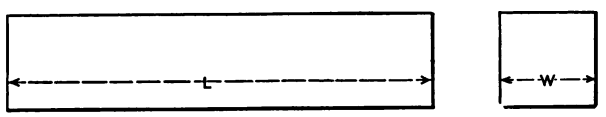


Fig. 2

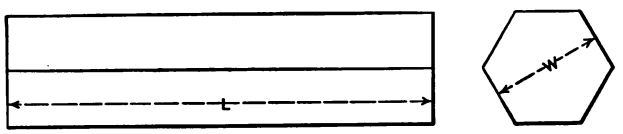


Fig. 3

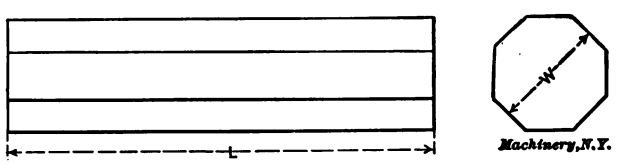


Fig. 4

one cubic inch of brass is 0.289 pound, and the weight of the given bar is, therefore, $68\frac{3}{4} \times 0.289 = 19.869$ pounds.

In order to find the weight of a hexagonal bar, as shown in Fig. 3, when the width across flats, W, the length L, and the weight per cubic inch of the material from which the bar is made, are known, the area of its end section must first be found so that the volume can be determined by multiplying this area by the length; when the width across flats, W, is given, this area equals $0.866 \times$ the square of the width across flats.

Assume that the weight is to be found of a hexagonal piece of machine steel bar stock 3 inches across flats, and 6 inches long. The volume of this piece equals then $0.866 \times 3^2 \times 6 = 0.866 \times 3 \times 3 \times 6 = 46.764$ cubic inches, and the weight equals $46.764 \times 0.283 = 13.234$ pounds. The factor 0.283 is the weight of one cubic inch of machine steel, as given in the table on page 90.

In order to find the weight of a piece of octagonal stock, as shown in Fig. 4, it is first necessary to find the area of the end section; when the width across flats, W, is given, this area equals $0.828 \times$ the square of the width across flats.

Assume that the weight of an octagonal piece of tool steel 4 inches across flats and 15 inches long is to be found. The volume

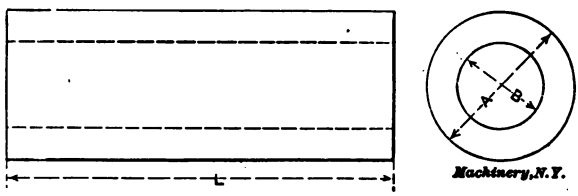


Fig. 5

of this piece then equals $0.828 \times 4^2 \times 15 = 0.828 \times 4 \times 4 \times 15 = 198.72$ cubic inches, and the weight equals $198.72 \times 0.283 = 56.238$ pounds. The factor 0.283 is the weight of one cubic inch of tool steel as given in the table at the beginning of this chapter.

The Weight of Castings. The weight of a casting can be calculated when the volume of the casting and the specific gravity or the weight per cubic inch of the material from which the casting is made, are known. If the volume is known in cubic inches, the volume is simply multiplied by the weight per cubic inch to obtain the weight of the casting.

The specific gravity of cast iron is 7.2 and the weight per cubic inch is 0.260; the specific gravity of brass is 8 and the weight per cubic inch is 0.289; the specific gravity of cast zinc is 6.86, and the weight per cubic inch 0.248; the specific gravity of gun metal bronze is 8.7 and the weight per cubic inch is 0.314.

With the constants above given, the problem of finding the weight of castings reduces itself to finding the volume of the casting. The multiplication by the weight per cubic inch of the material is then a simple matter.

Assume that it is required to find the weight of a hollow cast iron cylinder, as shown in Fig. 5, where the outside diameter is A, the inside or core diameter B, and the length L. To find the volume, first calculate the volume of a cylinder with the diameter A and the length L and then subtract from this the volume of the cylinder forming the core.

Assume that in a hollow cylinder as shown in Fig. 5, $A = 3$ inches, $B = 2$ inches, and $L = 8$ inches. The volume of a cylinder = 0.7854 × the square of the diameter × the height. The volume of a cylinder with 3 inches diameter and a height of 8 inches = $0.7854 \times 3^2 \times 8 = 0.7854 \times 3 \times 3 \times 8 = 56.5488$ cubic

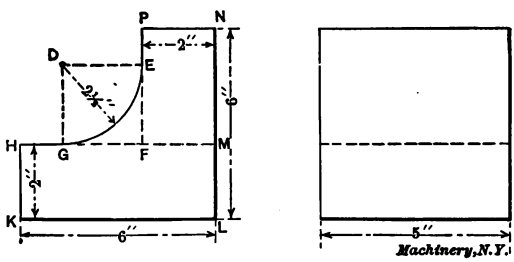


Fig. 6. Bracket or Knee

inches. From this is subtracted the volume of the cylinder forming the core, which has a diameter of 2 inches. The volume of this cylinder is $0.7854 \times 2^2 \times 8 = 25.1328$ cubic inches. This last volume subtracted from the volume 56.5488 gives us 31.416 cubic inches as the volume of the hollow cylinder (56.5488 − 25.1328 = 31.416). As the weight per cubic inch of cast iron is 0.260 pound, the total weight of the hollow cylinder will be $31.416 \times 0.260 = 8.168$ pounds.

If the cylinder had been cast from gun metal bronze instead of cast iron, the volume should be multiplied by 0.314, in order to find the weight.

If the outside diameter of a hollow cylinder is A, the inside diameter B, and the length L, the following formula may be used for finding the volume of the cylinder:

$$\text{Volume} = 0.7854 \times (A^2 - B^2) \times L$$

In Fig. 6 is shown a knee made from cast iron, all the necessary dimensions for calculating the weight being given. To calculate the volume of a casting of this shape, it is divided into prisms or

other simple geometric shapes, and the volume of each of the parts is found, after which these volumes are added together to find the total volume of the casting. The piece shown in Fig. 6 can be divided up into three parts, the volume of each of which can be calculated by simple means. One part has for base the rectangle *HMLK*, another the rectangle *PFMN*, and the base of the third is bounded by two straight lines *EF* and *FG* and the circular arc *EG*. The length of all the parts in this case equals the length of the casting, or 5 inches, as shown.

The area of the rectangle *HMLK* equals $6 \times 2 = 12$ square inches. This area multiplied by 5 gives us the volume of this part in cubic inches; $12 \times 5 = 60$ cubic inches.

The length of the line *NM* is 4 inches ($6 - 2 = 4$), and, therefore, the area of the rectangle *PFMN* is $4 \times 2 = 8$ square inches. This area multiplied by 5 gives us the volume of this section of the casting in cubic inches; $8 \times 5 = 40$ cubic inches.

It now remains to find the volume of the section having for base the area bounded by the two straight lines *EF* and *FG* and the circular arc *EG*. The area of the base is found by first finding the area of the square *DEFG* and subtracting from this area the area of the circular sector *DEG*. The area of the square is $2\frac{1}{2} \times 2\frac{1}{2} = 6\frac{1}{4}$ square inches. The area of the circular sector which is one-fourth of a complete circle is $\frac{2\frac{1}{2}^2 \times 3.1416}{4} = 4.909$ square inches. This subtracted from the area of the square equals 1.341 square inch ($6.25 - 4.909 = 1.341$). This is then the area of the base of the third part into which the casting is divided, and this area multiplied by 5 gives the volume of the third part of the casting ($1.341 \times 5 = 6.705$). Now adding the volumes of the three parts together we have $60 + 40 + 6.705 = 106.705$ cubic inches. This total volume multiplied by the weight per cubic inch of cast iron gives us the total weight: $106.705 \times 0.260 = 27.743$ pounds.

When the pattern for a casting contains no core-prints, but is in all respects an exact duplicate of the casting to be made, the weight of the casting may be approximately found by multiplying the weight of the pattern by a constant which varies for different kinds of woods used for the pattern.

CHAPTER XIII

USE OF TABLES OF SINES, COSINES, TANGENTS AND COTANGENTS

THE average mechanic usually looks upon the figuring of angles as something above his capacity; but simple cases of the figuring of angles from given formulas are often much easier than many ordinary shop problems in arithmetic which he successfully solves. All that is necessary is a table of sines, cosines, tangents, and cotangents; after having found the figures corresponding to a given angle from the table, the whole thing resolves itself into a case of simple multiplication or division.

Often, in technical papers, the reader will find himself confronted by such formulas as, for instance,

$$A = \frac{27}{\cos 36 \text{ deg.}}$$

Of course, it is impossible to figure out how much A is from this formula, unless the expression "cos 36 deg." (read: cosine of 36 degrees) can be transformed and expressed in plain figures. But if we know how much "cos 36 deg." is when expressed in plain figures, then we can immediately divide 27 by this value, and thus find the value of A. Suppose that A stands for the length of one side in a triangle and that the expression "cos 36 deg." equals 0.80901. Then,

$$A = \frac{27}{0.80901} = 33.37$$

The tables of sines, cosines, tangents, and cotangents simply serve the purpose of giving in figures the values of these expressions for different angles. The four expressions: sine, cosine, tangent, and cotangent, which are used to designate certain numerical values, to be found from the tables, are called the *functions of the angle.* These functions or numerical values equal a definite amount for each different angle. Tables will be found in this chapter which give the values referred to for all degrees and for

every ten minutes ($\frac{1}{6}$ of a degree). From these tables, when the angle is given, the angular function can be found, and when the function is given, the angle can be determined. The four expressions sine, cosine, tangent, and cotangent are abbreviated "sin." "cos," "tan," and "cot," respectively.

The tables of sines, cosines, etc., are read in the same way as a railroad time-table. At the top of Tables I and II the heading reads "Table of Sines," and at the bottom is the legend "Table of Cosines." At the top of Tables III and IV the heading reads "Table of Tangents," and at the bottom is the legend "Table of Cotangents." At the top of all the tables the heading of the extreme left-hand column reads "Deg." and the following columns are headed 0′, 10′, 20′, etc. At the bottom of the tables the same legends are placed under the columns, but reading from right to left.

When the sine or tangent of a given angle is to be found, first find the number of degrees in the extreme left-hand column in the respective tables, and then locate the number of minutes at the top of the table. Then follow the column, over which the number of minutes is given, downward until arriving at the figure in line with the given number of degrees. This figure is the numerical value of the sine or tangent, as the case may be, for the given angle. If the angle is given in even degrees with no minutes, the corresponding function will be found opposite the number of degrees in the column marked 0′ at the top.

The cosines and cotangents of angles are found in the same tables as the sines and tangents, but the tables in this case are read *from the bottom up*. The number of degrees is found in the extreme right-hand column and the number of minutes at the bottom of the columns.

If the number of minutes given is not an even multiple of 10, as 10′, 20′, 30′, etc., but 27′, for example, it is near enough for nearly all shop calculations to take the figures for the nearest number of minutes given, being in this case, for 30′.

Examples of the Use of the Tables. *Example* 1. Find from the tables the sine of 56 degrees, or, as it is written in formulas, sin 56°. The "sines" are found by reading Tables I and II from the top; the number of degrees, 56, is found in Table II in the left-hand column, and opposite 56 in the column 0′, read off 0.82903.

I. TABLE OF SINES

Read degrees in left-hand column and minutes at top

Example: sin 7° 10′ = .12475

Deg.	0′	10′	20′	30′	40′	50′	60′	
0	.00000	.00291	.00581	.00872	.01163	.01454	.01745	89
1	.01745	.02036	.02326	.02617	.02908	.03199	.03489	88
2	.03489	.03780	.04071	.04361	.04652	.04943	.05233	87
3	.05233	.05524	.05814	.06104	.06395	.06685	.06975	86
4	.06975	.07265	.07555	.07845	.08135	.08425	.08715	85
5	.08715	.09005	.09295	.09584	.09874	.10163	.10452	84
6	.10452	.10742	.11031	.11320	.11609	.11898	.12186	83
7	.12186	.12475	.12764	.13052	.13341	.13629	.13917	82
8	.13917	.14205	.14493	.14780	.15068	.15356	.15643	81
9	.15643	.15930	.16217	.16504	.16791	.17078	.17364	80
10	.17364	.17651	.17937	.18223	.18509	.18795	.19080	79
11	.19080	.19366	.19651	.19936	.20221	.20506	.20791	78
12	.20791	.21075	.21359	.21644	.21927	.22211	.22495	77
13	.22495	.22778	.23061	.23344	.23627	.23909	.24192	76
14	.24192	.24474	.24756	.25038	.25319	.25600	.25881	75
15	.25881	.26162	.26443	.26723	.27004	.27284	.27563	74
16	.27563	.27843	.28122	.28401	.28680	.28958	.29237	73
17	.29237	.29515	.29793	.30070	.30347	.30624	.30901	72
18	.30901	.31178	.31454	.31730	.32006	.32281	.32556	71
19	.32556	.32831	.33106	.33380	.33654	.33928	.34202	70
20	.34202	.34475	.34748	.35020	.35293	.35565	.35836	69
21	.35836	.36108	.36379	.36650	.36920	.37190	.37460	68
22	.37460	.37730	.37999	.38268	.38536	.38805	.39073	67
23	.39073	.39340	.39607	.39874	.40141	.40407	.40673	66
24	.40673	.40939	.41204	.41469	.41733	.41998	.42261	65
25	.42261	.42525	.42788	.43051	.43313	.43575	.43837	64
26	.43837	.44098	.44359	.44619	.44879	.45139	.45399	63
27	.45399	.45658	.45916	.46174	.46432	.46690	.46947	62
28	.46947	.47203	.47460	.47715	.47971	.48226	.48481	61
29	.48481	.48735	.48989	.49242	.49495	.49747	.50000	60
30	.50000	.50251	.50503	.50753	.51004	.51254	.51503	59
31	.51503	.51752	.52001	.52249	.52497	.52745	.52991	58
32	.52991	.53238	.53484	.53730	.53975	.54219	.54463	57
33	.54463	.54707	.54950	.55193	.55436	.55677	.55919	56
34	.55919	.56160	.56400	.56640	.56880	.57119	.57357	55
35	.57357	.57595	.57833	.58070	.58306	.58542	.58778	54
36	.58778	.59013	.59248	.59482	.59715	.59948	.60181	53
37	.60181	.60413	.60645	.60876	.61106	.61336	.61566	52
38	.61566	.61795	.62023	.62251	.62478	.62705	.62932	51
39	.62932	.63157	.63383	.63607	.63832	.64055	.64278	50
40	.64278	.64501	.64723	.64944	.65165	.65386	.65605	49
41	.65605	.65825	.66043	.66262	.66479	.66696	.66913	48
42	.66913	.67128	.67344	.67559	.67773	.67986	.68199	47
43	.68199	.68412	.68624	.68835	.69046	.69256	.69465	46
44	.69465	.69674	.69883	.70090	.70298	.70504	.70710	45
	60′	50′	40′	30′	20′	10′	0′	Deg.

TABLE OF COSINES

Read degrees in right-hand column and minutes at bottom

Example: cos 56° 20′ = .55436

II. TABLE OF SINES

Read degrees in left-hand column and minutes at top

Example: sin 56° 20′ = .83227

Deg.	0′	10′	20′	30′	40′	50′	60′	
45	.70710	.70916	.71120	.71325	.71528	.71731	.71934	44
46	.71934	.72135	.72336	.72537	.72737	.72936	.73135	43
47	.73135	.73333	.73530	.73727	.73923	.74119	.74314	42
48	.74314	.74508	.74702	.74895	.75088	.75279	.75471	41
49	.75471	.75661	.75851	.76040	.76229	.76417	.76604	40
50	.76604	.76791	.76977	.77162	.77347	.77531	.77714	39
51	.77714	.77897	.78079	.78260	.78441	.78621	.78801	38
52	.78801	.78979	.79157	.79335	.79512	.79688	.79863	37
53	.79863	.80038	.80212	.80385	.80558	.80730	.80901	36
54	.80901	.81072	.81242	.81411	.81580	.81748	.81915	35
55	.81915	.82081	.82247	.82412	.82577	.82740	.82903	34
56	.82903	.83066	.83227	.83388	.83548	.83708	.83867	33
57	.83867	.84025	.84182	.84339	.84495	.84650	.84804	32
58	.84804	.84958	.85111	.85264	.85415	.85566	.85716	31
59	.85716	.85866	.86014	.86162	.86310	.86456	.86603	30
60	.86602	.86747	.86892	.87035	.87178	.87320	.87462	29
61	.87462	.87602	.87742	.87881	.88020	.88157	.88294	28
62	.88294	.88430	.88566	.88701	.88835	.88968	.89100	27
63	.89100	.89232	.89363	.89493	.89622	.89751	.89879	26
64	.89879	.90006	.90132	.90258	.90383	.90507	.90630	25
65	.90630	.90753	.90875	.90996	.91116	.91235	.91354	24
66	.91354	.91472	.91589	.91706	.91821	.91936	.92050	23
67	.92050	.92163	.92276	.92388	.92498	.92609	.92718	22
68	.92718	.92827	.92934	.93041	.93148	.93253	.93358	21
69	.93358	.93461	.93565	.93667	.93768	.93869	.93969	20
70	.93969	.94068	.94166	.94264	.94360	.94456	.94551	19
71	.94551	.94646	.94739	.94832	.94924	.95015	.95105	18
72	.95105	.95195	.95283	.95371	.95458	.95545	.95630	17
73	.95630	.95715	.95799	.95882	.95964	.96045	.96126	16
74	.96126	.96205	.96284	.96363	.96440	.96516	.96592	15
75	.96592	.96667	.96741	.96814	.96887	.96958	.97029	14
76	.97029	.97099	.97168	.97237	.97304	.97371	.97437	13
77	.97437	.97502	.97566	.97629	.97692	.97753	.97814	12
78	.97814	.97874	.97934	.97992	.98050	.98106	.98162	11
79	.98162	.98217	.98272	.98325	.98378	.98429	.98480	10
80	.98480	.98530	.98580	.98628	.98676	.98722	.98768	9
81	.98768	.98813	.98858	.98901	.98944	.98985	.99026	8
82	.99026	.99066	.99106	.99144	.99182	.99218	.99254	7
83	.99254	.99289	.99323	.99357	.99389	.99421	.99452	6
84	.99452	.99482	.99511	.99539	.99567	.99593	.99619	5
85	.99619	.99644	.99668	.99691	.99714	.99735	.99756	4
86	.99756	.99776	.99795	.99813	.99830	.99847	.99863	3
87	.99862	.99877	.99891	.99904	.99917	.99928	.99939	2
88	.99939	.99948	.99957	.99965	.99972	.99979	.99984	1
89	.99984	.99989	.99993	.99996	.99998	.99999	1.0000	0
	60′	50′	40′	30′	20′	10′	0′	Deg.

TABLE OF COSINES

Read degrees in right-hand column and minutes at bottom

Example: cos 7° 10′ = .99218

III. TABLE OF TANGENTS

Read degrees in left-hand column and minutes at top

Example: tan 7° 10′ = .12573

Deg.	0′	10′	20′	30′	40′	50′	60′	
0	.00000	.00290	.00581	.00872	.01163	.01454	.01745	89
1	.01745	.02036	.02327	.02618	.02909	.03200	.03492	88
2	.03492	.03783	.04074	.04366	.04657	.04949	.05240	87
3	.05240	.05532	.05824	.06116	.06408	.06700	.06992	86
4	.06992	.07285	.07577	.07870	.08162	.08455	.08748	85
5	.08748	.09042	.09335	.09628	.09922	.10216	.10510	84
6	.10510	.10804	.11099	.11393	.11688	.11983	.12278	83
7	.12278	.12573	.12869	.13165	.13461	.13757	.14054	82
8	.14054	.14350	.14647	.14945	.15242	.15540	.15838	81
9	.15838	.16136	.16435	.16734	.17033	.17332	.17632	80
10	.17632	.17932	.18233	.18533	.18834	.19136	.19438	79
11	.19438	.19740	.20042	.20345	.20648	.20951	.21255	78
12	.21255	.21559	.21864	.22169	.22474	.22780	.23086	77
13	.23086	.23393	.23700	.24007	.24315	.24624	.24932	76
14	.24932	.25242	.25551	.25861	.26172	.26483	.26794	75
15	.26794	.27106	.27419	.27732	.28046	.28360	.28674	74
16	.28674	.28989	.29305	.29621	.29938	.30255	.30573	73
17	.30573	.30891	.31210	.31529	.31850	.32170	.32492	72
18	.32492	.32813	.33136	.33459	.33783	.34107	.34432	71
19	.34432	.34758	.35084	.35411	.35739	.36067	.36397	70
20	.36397	.36726	.37057	.37388	.37720	.38053	.38386	69
21	.38386	.38720	.39055	.39391	.39727	.40064	.40402	68
22	.40402	.40741	.41080	.41421	.41762	.42104	.42447	67
23	.42447	.42791	.43135	.43481	.43827	.44174	.44522	66
24	.44522	.44871	.45221	.45572	.45924	.46277	.46630	65
25	.46630	.46985	.47341	.47697	.48055	.48413	.48773	64
26	.48773	.49133	.49495	.49858	.50221	.50586	.50952	63
27	.50952	.51319	.51687	.52056	.52427	.52798	.53170	62
28	.53170	.53544	.53919	.54295	.54672	.55051	.55430	61
29	.55430	.55811	.56193	.56577	.56961	.57347	.57735	60
30	.57735	.58123	.58513	.58904	.59297	.59690	.60086	59
31	.60086	.60482	.60880	.61280	.61680	.62083	.62486	58
32	.62486	.62892	.63298	.63707	.64116	.64528	.64940	57
33	.64940	.65355	.65771	.66188	.66607	.67028	.67450	56
34	.67450	.67874	.68300	.68728	.69157	.69588	.70020	55
35	.70020	.70455	.70891	.71329	.71769	.72210	.72654	54
36	.72654	.73099	.73546	.73996	.74447	.74900	.75355	53
37	.75355	.75812	.76271	.76732	.77195	.77661	.78128	52
38	.78128	.78598	.79069	.79543	.80019	.80497	.80978	51
39	.80978	.81461	.81946	.82433	.82923	.83415	.83910	50
40	.83910	.84406	.84906	.85408	.85912	.86419	.86928	49
41	.86928	.87440	.87955	.88472	.88992	.89515	.90040	48
42	.90040	.90568	.91099	.91633	.92169	.92709	.93251	47
43	.93251	.93796	.94345	.94896	.95450	.96008	.96568	46
44	.96568	.97132	.97699	.98269	.98843	.99419	1.0000	45
	60′	50′	40′	30′	20′	10′	0′	Deg.

TABLE OF COTANGENTS

Read degrees in right-hand column and minutes at bottom

Example: cot 56° 20′ = .66607

IV. TABLE OF TANGENTS

Read degrees in left-hand column and minutes at top

Example: tan 56° 20′ = 1.5013

Deg.	0′	10′	20′	30′	40′	50′	60′	
45	1.0000	1.0058	1.0117	1.0176	1.0235	1.0295	1.0355	44
46	1.0355	1.0415	1.0476	1.0537	1.0599	1.0661	1.0723	43
47	1.0723	1.0786	1.0849	1.0913	1.0977	1.1041	1.1106	42
48	1.1106	1.1171	1.1236	1.1302	1.1369	1.1436	1.1503	41
49	1.1503	1.1571	1.1639	1.1708	1.1777	1.1847	1.1917	40
50	1.1917	1.1988	1.2059	1.2131	1.2203	1.2275	1 2349	39
51	1.2349	1.2422	1.2496	1.2571	1.2647	1.2723	1.2799	38
52	1.2799	1.2876	1.2954	1.3032	1.3111	1.3190	1.3270	37
53	1.3270	1.3351	1.3432	1.3514	1.3596	1.3680	1.3763	36
54	1.3763	1.3848	1.3933	1.4019	1.4106	1.4193	1.4281	35
55	1.4281	1.4370	1.4459	1.4550	1.4641	1.4733	1.4825	34
56	1.4825	1.4919	1.5013	1.5108	1.5204	1.5301	1.5398	33
57	1.5398	1.5497	1.5596	1.5696	1.5798	1.5900	1.6003	32
58	1.6003	1.6107	1.6212	1.6318	1.6425	1.6533	1.6642	31
59	1.6642	1.6753	1.6864	1.6976	1.7090	1.7204	1.7320	30
60	1.7320	1.7437	1.7555	1.7674	1.7795	1.7917	1.8040	29
61	1.8040	1.8164	1.8290	1.8417	1.8546	1.8676	1.8807	28
62	1.8807	1.8940	1.9074	1.9209	1.9347	1.9485	1.9626	27
63	1.9626	1.9768	1.9911	2.0056	2.0203	2.0352	2.0503	26
64	2.0503	2.0655	2.0809	2.0965	2.1123	2.1283	2.1445	25
65	2.1445	2.1609	2.1774	2.1943	2.2113	2.2285	2.2460	24
66	2.2460	2.2637	2.2816	2.2998	2.3182	2.3369	2.3558	23
67	2.3558	2.3750	2.3944	2.4142	2.4342	2.4545	2.4750	22
68	2.4750	2.4959	2.5171	2.5386	2.5604	2.5826	2.6050	21
69	2.6050	2.6279	2.6510	2.6746	2.6985	2.7228	2.7474	20
70	2.7474	2.7725	2.7980	2.8239	2.8502	2.8770	2.9042	19
71	2.9042	2.9318	2.9600	2.9886	3.0178	3.0474	3.0776	18
72	3.0776	3.1084	3.1397	3.1715	3.2040	3.2371	3.2708	17
73	3.2708	3.3052	3.3402	3.3759	3.4123	3.4495	3.4874	16
74	3.4874	3.5260	3.5655	3.6058	3.6470	3.6890	3.7320	15
75	3.7320	3.7759	3.8208	3.8667	3.9136	3.9616	4.0107	14
76	4.0107	4.0610	4.1125	4.1653	4.2193	4.2747	4.3314	13
77	4.3314	4.3896	4.4494	4.5107	4.5736	4.6382	4.7046	12
78	4.7046	4.7728	4.8430	4.9151	4.9894	5.0658	5.1445	11
79	5.1445	5.2256	5.3092	5.3955	5.4845	5.5763	5.6712	10
80	5.6712	5.7693	5.8708	5.9757	6.0844	6.1970	6.3137	9
81	6.3137	6.4348	6.5605	6.6911	6.8269	6.9682	7.1153	8
82	7.1153	7.2687	7.4287	7.5957	7.7703	7.9530	8.1443	7
83	8.1443	8.3449	8.5555	8.7768	9.0098	9.2553	9.5143	6
84	9.5143	9.7881	10.078	10.385	10.711	11.059	11.430	5
85	11.430	11.826	12.250	12.706	13.196	13.726	14.300	4
86	14.300	14.924	15.604	16.349	17.169	18.075	19.081	3
87	19.081	20.205	21.470	22.904	24.541	26.431	28.636	2
88	28.636	31.241	34.367	38.188	42.964	49.103	57.290	1
89	57.290	68.750	85.939	114.58	171.88	343.77	∞	0
	60′	50′	40′	30′	20′	10′	0′	Deg.

TABLE OF COTANGENTS

Read degrees in right-hand column and minutes at bottom

Example: cot 7° 10′ = 7.9530

Example 2. Find sin 56° 20′. In the column marked 20′ at the top, follow downwards until opposite 56 in the left-hand column. The value 0.83227 is the sine of 56° 20′.

Example 3. Find cos 36° 20′. To find the cosines, read the tables from the bottom, and locate 36 in the right-hand column in Table II. Then follow the column marked 20′ at the bottom upwards until opposite 36, and read off 0.80558.

Example 4. Find tan 56° 40′. The tangents are found in Tables III and IV by locating the number of degrees in the left-hand column and reading the value in the column under the specified number of minutes. In Table IV then we find tan 56° 40′ to be 1.5204.

Example 5. Find the cotangent of 56° 40′. Read the tables from the bottom, locating 56 in the right-hand column, and find the required value in line with this figure in the column marked 40′ at the bottom. Thus, cot 56° 40′ = 0.65771.

Example 6. Find sin 20° 48′. For shop calculations it is almost always near enough to find the value of the angular functions for the nearest 10 minutes. Therefore in this case find sin 20° 50′, which is 0.35565.

Example 7. The sine for a certain angle, which may be called α, equals 0.53238. Find the angle. In the body of the tables of sines find the number 0.53238. It will be seen that this number is opposite 32 degrees and in the column headed 10′ at the top. The angle α, therefore, equals 32° 10′.

Example 8. Cot β = 0.77195. Find β. The cotangents are read from the bottom in Tables III and IV. The value 0.77195 is located opposite 52 in the right-hand column and in the column marked 20′ at the bottom. Angle β, then, is 52° 20′.

Example 9. Sin β = 0.31190. Find β. It will be found that the value 0.31190 is not given in the table of sines; the nearest value in the table is 0.31178. For shop calculations it is near enough to consider the angle β equal to the angle corresponding to this latter value; the angle then is 18° 10′.

Right-angled Triangles. In right-angled triangles the remaining sides and angles can be found when either two sides, or one side and one of the acute angles, are known. As previously mentioned, the side opposite the right angle, or side a in Fig. 1, is called the *hypotenuse.* Side b is the side *adjacent* to angle C and side c is the side *opposite* to the same angle. In the same

way, c is the side *adjacent* to angle B, and b is the side *opposite* angle B.

The problems in right-angled triangles may be divided into five classes, for which the following formulas are given:

1. The hypotenuse and one of the sides forming the right angle are given. Call the hypotenuse a, and the other given side b. Then (see Fig. 1):

$$c = \sqrt{a^2 - b^2} \qquad \sin B = \frac{b}{a} \qquad C = 90° - B$$

2. The two sides forming the right angle are given. Call these two sides b and c. Then (see Fig. 1):

$$a = \sqrt{b^2 + c^2} \qquad \tan B = \frac{b}{c} \qquad C = 90° - B$$

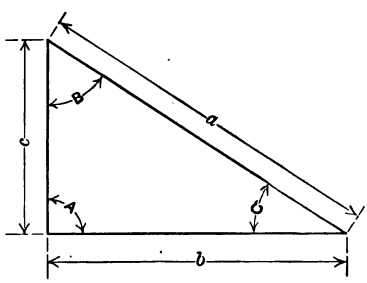


Fig. 1. Right-angled Triangle

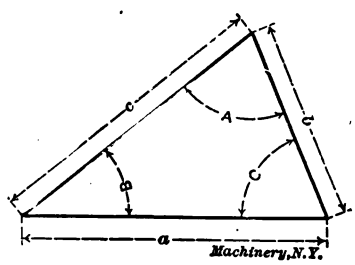


Fig. 2. Oblique-angled Triangle

3. The hypotenuse and one of the acute angles are given. Call the hypotenuse a and the given angle B. Then (see Fig. 1):

$$C = 90° - B \qquad b = a \times \sin B \qquad c = a \times \cos B$$

4. One acute angle and its adjacent side are given. Call the given angle B and its adjacent side c. Then (see Fig. 1):

$$C = 90° - B \qquad a = \frac{c}{\cos B} \qquad b = c \times \tan B$$

5. One acute angle and its opposite side are given. Call the given angle B and the side opposite it b. Then (see Fig. 1):

$$C = 90° - B \qquad a = \frac{b}{\sin B} \qquad c = b \times \cot B$$

Formulas for Solving Oblique Triangles. Below are given a summary of all the generally required formulas, and the methods

of procedure in solving oblique triangles. In all the formulas reference is made to Fig. 2, in which the sides and angles are given the same names as in the formulas below.

1. When two angles and one side are given, call the given side a, the angle opposite it A, and the other angle B. Then if A is known:

$$C = 180° - (A + B) \qquad b = \frac{a \times \sin B}{\sin A} \qquad c = \frac{a \times \sin C}{\sin A}$$

$$\text{Area} = \frac{a \times b \times \sin C}{2}$$

If B and C are given, but not A, then $A = 180° - (B + C)$, the other formulas being as above.

2. When two sides and the included angle are given, call the given sides a and b and the given angle between them C. Then:

$$\tan A = \frac{a \times \sin C}{b - a \times \cos C} \qquad B = 180° - (A + C) \qquad c = \frac{a \times \sin C}{\sin A}$$

$$\text{Area} = \frac{a \times b \times \sin C}{2}$$

3. When two sides and the angle opposite one of the sides are given, call the given angle A, the side opposite it a and the other given side b. Then:

$$\sin B = \frac{b \times \sin A}{a} \qquad C = 180° - (A + B) \qquad c = \frac{a \times \sin C}{\sin A}$$

$$\text{Area} = \frac{a \times b \times \sin C}{2}$$

4. When the three sides of a triangle are given, call them a, b and c and the angles opposite them A, B and C respectively. Then:

$$\cos A = \frac{b^2 + c^2 - a^2}{2 \times b \times c} \qquad \sin B = \frac{b \times \sin A}{a} \qquad C = 180° - (A + B)$$

$$\text{Area} = \frac{a \times b \times \cos C}{2}$$

CHAPTER XIV

SQUARE AND CUBE ROOTS

THE square of a number is the product of that number multiplied by itself. The square of 2 is $2 \times 2 = 4$, and the square of 10 is $10 \times 10 = 100$; similarly the square of 177 is $177 \times 177 = 31{,}329$. Instead of writing 2×2 for the square of 2, it is often written 2^2, which is read *two square*, and means that 2 is multiplied by 2. In the same way 128^2 means 128×128. The small figure $(^2)$ in these expressions is called *exponent.*

The square root of a number is that number which, when multiplied by itself, will give a product equal to the given number. Thus, the square root of 4 is 2, because 2 multiplied by itself gives 4. The square root of 25 is 5; of 36, 6, etc. We may say that the square root is the reverse of the square, so that if the square of 24 is 576, then the square root of 576 is 24. The mathematical sign for the square root is $\sqrt[2]{\ }$, but the *index figure* $(^2)$ is generally left out, making the square-root sign simply $\sqrt{\ }$, thus:

$\sqrt{4} = 2$ (the square root of four equals two),

$\sqrt{100} = 10$ (the square root of one hundred equals ten).

The operation of finding the square root of a given number is called *extracting* the square root.

Assume that the square root of 119,716 is to be found. Write the number as below, leaving space for the figures of the root as shown. Beginning at the unit figure (the last figure at the right of a whole number), point off the number into periods of two figures each. Should there be an odd number of figures in the given number, the last period to the left will, of course, have only one figure.

11'97'16 | Space for root.

Now find the greatest whole number the square of which does not exceed the value of the figures in the left-hand period (11), and write this number as the first figure of the root. In the example this number is 3, the square of which is 9. Subtract this

square from the left-hand period, and move down the next period of two figures and annex it to the remainder, thus:

$$\begin{array}{rl} & 11'97'16 \mid \underline{3} \\ 3 \times 3 = & \underline{9} \\ & 297 \end{array}$$

Now multiply the figure of the root obtained by the constant 20 which is always used when extracting the square root by this method ($3 \times 20 = 60$), and find how many times this product is contained in the number 297. This gives us a trial figure for the second figure of the root; 60 is contained 4 whole times in 297, and 4 is, therefore, placed as the next figure of the root.

$$\begin{array}{rl} & 11'97'16 \mid \underline{34} \\ 3 \times 3 = & \underline{9} \\ 3 \times 20 = 60 & 297 \end{array}$$

Now subtract from 297 the product of 60 plus the figure of the root just obtained (4), multiplied by the same figure (4); $(60 + 4) \times 4 = 256$. If this product were larger than 297 it would indicate that the trial figure is too large, and a figure one unit smaller should be used.

Then move down the next period of two figures and annex it to the remainder.

$$\begin{array}{rl} & 11'97'16 \mid \underline{34} \\ 3 \times 3 = & 9 \\ 3 \times 20 = 60 & \underline{297} \\ (60 + 4) \times 4 = & \underline{256} \\ & 4116 \end{array}$$

Now multiply the figures of the root thus far obtained by 20; $(34 \times 20 = 680)$, and find how many times this product is contained in 4116. This gives us a trial figure for the third figure of the root; 680 is contained 6 times in 4116, and 6 is therefore placed as the third figure of the root. Then subtract from 4116 the product of 680 plus the figure of the root just obtained (6), multiplied by the same figure (6).

$$\begin{array}{rl} & 11'97'16 \mid \underline{346} \\ 3 \times 3 = & 9 \\ 3 \times 20 = 60 & \underline{297} \\ (60 + 4) \times 4 = & \underline{256} \\ 34 \times 20 = 680 & 4116 \\ (680 + 6) \times 6 = & \underline{4116} \end{array}$$

If, as in the present case, this last subtraction leaves no remainder, and if there are no more periods of figures to move down from the given number, the obtained root 346 is the exact square root of 119,716.

If there is a remainder when the last period of figures has been moved down, place a decimal point after the figures already obtained in the root, annex two ciphers (00) to the remainder, multiply the number so far obtained in the root by 20, and proceed as before until a sufficient number of decimals have been obtained to give the root with sufficient accuracy.

Example:

	1'25 | 11.18
$1 \times 1 =$	1
$1 \times 20 = 20$	25
$(20 + 1) \times 1 =$	21
$11 \times 20 = 220$	400
$(220 + 1) \times 1 =$	221
$111 \times 20 = 2220$	17900
$(2220 + 8) \times 8 =$	17824

It will be seen from the calculation that when multiplying by the constant 20, the decimal point is disregarded, and the figures obtained in the root considered as a whole number. The decimal point must, however, be placed in the root as already explained before annexing the two first ciphers (not in the given number) to the remainder, in order to give a correct value to the root.

When extracting the square root of a decimal fraction, or when the square root of a whole number and a decimal is required, always point off *both* the whole number and the decimal in periods of two figures each, *beginning at the decimal point,* thus:

2'17'63.56'78'5

If the number of decimal places is not an even number, the period to the right will have only one figure instead of two. By placing a cipher after the decimal in such cases, the last period is made complete without changing the value of the number, thus:

2'17'63.56'78'50

It should be borne in mind that the pointing off of periods of two figures each should always be begun at the decimal point, both for the whole numbers and for the decimals. Thus, for in-

stance, the pointing off in the first line below is correct, while the pointing off in the second line is incorrect:

Correctly pointed off:	0.76'34'5	3'26.75'4
Incorrectly pointed off:	0.7'63'45	32'6.7'54

When extracting the square root of a decimal fraction, the decimal point is placed in the root when the first period of decimals is moved down.

Example:

$$\begin{array}{rl} & 5.71'21 \mid \underline{2.39} \\ 2 \times 2 = & \underline{4} \\ 2 \times 20 = 40 & 171 \\ (40 + 3) \times 3 = & \underline{129} \\ 23 \times 20 = 460 & 4221 \\ (460 + 9) \times 9 = & \underline{4221} \end{array}$$

When it is found that the next figure in the root is a cipher, place it as usual in the root, and move down the next period of two figures, in all other respects following the procedure already explained.

Example:

$$\begin{array}{rl} & 9'12'04 \mid \underline{302} \\ 3 \times 3 = & \underline{9} \\ \left.\begin{array}{l} 3 \times 20 = 60 \\ 30 \times 20 = 600 \end{array}\right\} & 1204 \\ (600 + 2) \times 2 = & \underline{1204} \end{array}$$

The square root of a common fraction may be obtained by extracting the square root of both numerator and denominator, thus

$$\sqrt{\frac{25}{49}} = \frac{\sqrt{25}}{\sqrt{49}} = \frac{5}{7}$$

When the terms of the fraction are not perfect squares (squares of whole numbers), it is preferable to change the common fraction to a decimal fraction, and extract the square root of this.

When there is no remainder after all the periods of figures in the given number have been moved down, and the last figure of the root found, the calculation may be proved by multiplying the root by itself, in which case the product must equal the number given, of which the square root has been extracted. If there is a remainder, the figures obtained do not represent the exact root, but a close approximation; if this approximate root is multiplied

by itself, the product should *very nearly* equal the given number; if not, an error has been made.

The cube of a number is the product obtained if the number itself is repeated as a factor three times. The cube of 2 is $2 \times 2 \times 2 = 8$, and the cube of 12 is $12 \times 12 \times 12 = 1{,}728$. Instead of writing $2 \times 2 \times 2$ for the cube of 2, it is often written 2^3, which is read "two cube." In the same way 128^3 means $128 \times 128 \times 128$. The small figure (^{3}) in these expressions is called *exponent.* An expression of the form 18^3 may be read the "third power of 18."

In the same way as square root means the reverse of square, so cube root means the reverse of cube; that is, the cube root of a given number is the number which, if repeated as factor three times, would give the number given. Thus the cube root of 27 is 3, because $3 \times 3 \times 3 = 27$. If the cube of 15 is 3,375, then the cube root of 3,375 is, of course, 15. The mathematical sign for the cube root is $\sqrt[3]{\ }$, thus:

$\sqrt[3]{64} = 4$ (the cube root of sixty-four equals four),

$\sqrt[3]{4096} = 16$ (the cube root of four thousand ninety-six equals sixteen).

In the case of all roots, except the square root, the index, or the small figure in the radical sign ($\sqrt{\ }$), must be given.

Assume that the cube root of 80,621,568 is to be found. Write the number as below, leaving space for the figures of the root as shown. Beginning at the unit figure (the last figure at the right of a whole number), point off the number into periods of *three* figures each. According to the total number of figures in the given number, the last period to the left will, of course, have one, two or three figures.

80'621'568 | Space for root.

Now find the greatest whole number, the cube of which does not exceed the value of the figures in the left-hand period (80), and write this number as the first figure in the root. The cube of 4 is 64 ($4 \times 4 \times 4 = 64$), and the cube of 5 is 125 ($5 \times 5 \times 5 = 125$). Hence 4 is the greatest whole number, the cube of which does not exceed 80, and 4, therefore, is the first figure of the root. Subtract the cube of 4 from the left-hand period and move down the next period of three figures, and annex it to the remainder.

80'621'568 | 4

$4 \times 4 \times 4 = 64$

16621

Now multiply the square of the figure in the root by the constant 300, which is always used when extracting the cube root by this method ($4^2 \times 300 = 4 \times 4 \times 300 = 4{,}800$), and find how many times this product is contained in the number 16,621. This gives us a trial figure for the second figure of the root; 4,800 is contained three whole times in 16,621, and 3 is therefore placed as the next figure of the root:

$$\begin{array}{rr} & 80'621'568 \mid 43 \\ 4 \times 4 \times 4 = & 64 \\ 4^2 \times 300 = 4{,}800 & \overline{16621} \end{array}$$

Now subtract from 16,621 the *sum* of the following products:

1. The square of the figure or figures already obtained in the root, excepting the last one, multiplied by 300, and this product multiplied by the figure just obtained in the root, thus:

$$4^2 \times 300 \times 3 = 16 \times 300 \times 3 = 14{,}400$$

2. The figure or figures already obtained in the root, excepting the last one, multiplied by 30, and this product multiplied by the square of the last figure obtained, thus:

$$4 \times 30 \times 3^2 = 4 \times 30 \times 9 = 1{,}080$$

3. The cube of the last figure obtained, thus:

$$3^3 = 3 \times 3 \times 3 = 27$$

The method followed will be understood by studying the example and comparing the different quantities with the worded explanations just given. If the sum of these various products is larger than 16,621, it indicates that the trial figure is too large, and a figure one unit smaller should be used.

Now move down the next period of three figures, and annex it to the remainder.

$$\begin{array}{rr} & 80'621'568 \mid 43 \\ 4 \times 4 \times 4 = & 64 \\ 4^2 \times 300 = 4{,}800 & \overline{16621} \\ 4^2 \times 300 \times 3 + 4 \times 30 \times 3^2 + 3^3 = & 15507 \\ & \overline{1114568} \end{array}$$

Multiply the square of the figures of the root thus far obtained by 300 ($43^2 \times 300 = 43 \times 43 \times 300 = 554{,}700$), and find how many times this product is contained in 1,114,568. This gives a trial figure for the third figure of the root; 554,700 is contained two

times in 1,114,568, and 2 is therefore placed as the third figure of the root. Now subtract from 1,114,568 a sum made up of the three products previously given, and shown in the example below:

$$\begin{array}{rl} & 80'621'568 \mid 432 \\ 4 \times 4 \times 4 = & 64 \\ 4^2 \times 300 = 4{,}800 & 16621 \\ 4^2 \times 300 \times 3 + 4 \times 30 \times 3^2 + 3^3 = & 15507 \\ 43^2 \times 300 = 554{,}700 & 1114568 \\ 43^2 \times 300 \times 2 + 43 \times 30 \times 2^2 + 2^3 = & 1114568 \end{array}$$

If, as in the present case, this last subtraction leaves no remainder, and if there are no more periods of figures to move down from the given number, the obtained root 432 is the exact cube root of 80,621,568.

If there is a remainder when the last period of three figures has been moved down, place a decimal point after the figures already obtained in the root, annex three ciphers (000) to the remainder, multiply the square of the number thus far obtained in the root by 300, and proceed as before until a sufficient number of decimals have been obtained to give the root with sufficient accuracy.

Example:

$$\begin{array}{rl} & 1'816 \mid 12.2 \\ 1 \times 1 \times 1 = & 1 \\ 1^2 \times 300 = 300 & 816 \\ 1^2 \times 300 \times 2 + 1 \times 30 \times 2^2 + 2^3 = & 728 \\ 12^2 \times 300 = 43{,}200 & 88000 \\ 12^2 \times 300 \times 2 + 12 \times 30 \times 2^2 + 2^3 = & 87848 \end{array}$$

It should be noted in these calculations that when squaring the figures thus far obtained in the root, and multiplying by the constant 300, the decimal point is disregarded and the figures obtained in the root considered as a whole number. The decimal point must, however, be placed in the root as already explained, before annexing the first three ciphers (not in the given number) to the remainder, in order to give a correct value of the root.

When the cube root of a number containing a whole number and a decimal is required, always point off *both* the whole number and the decimal in periods of three figures each, *beginning at the decimal point*, thus:

83'675'731.563'75

If the number of decimal places is not evenly divisible by three, the period to the right will have only one or two figures instead of

three. By placing one or two ciphers after the decimal in such cases, the last period is made complete without changing the value of the number, thus:

83'675'731.563'750

It should be borne in mind that the pointing off of periods of three figures each should always be begun at the decimal point, both for the whole number and for the decimals. Thus, for instance, the pointing off in the first line below is correct while the pointing off in the second line is incorrect:

Correctly pointed off:	0.765'354'3	2'765.354'2
Incorrectly pointed off:	0.7'653'543	27'65.3'542

When extracting the cube root of a decimal fraction, the decimal point is placed in the root when the first period of decimals is moved down.

When it is found that the next figure in the root is a cipher, place it as usual in the root and move down the next period of three figures, in all other respects following the procedure already explained.

The cube root of a common fraction may be obtained by extracting the cube root of both the numerator and denominator, thus

$$\sqrt[3]{\frac{27}{1000}} = \frac{\sqrt[3]{27}}{\sqrt[3]{1000}} = \frac{3}{10}$$

When the terms of the fraction are not perfect cubes (cubes of whole numbers), it is preferable to change the common fraction to a decimal fraction and then extract the cube root.

When there is no remainder after all the periods of figures in the given number have been moved down, and the last figure of the root found, the calculation may be proved by repeating the root as a factor three times, in which case the product must equal the number given, of which the cube root has been extracted. If there is a remainder, the figures obtained do not represent the exact root, but a close approximation. If this approximate root is repeated as a factor three times the product should *very nearly* equal the given number; if not, an error has been made.

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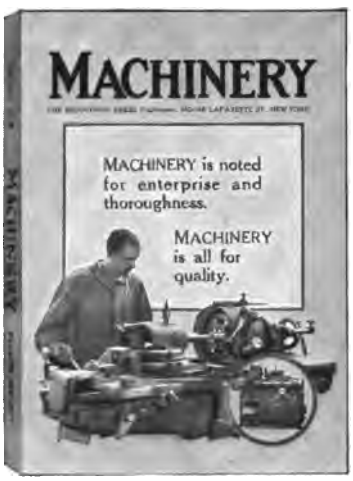

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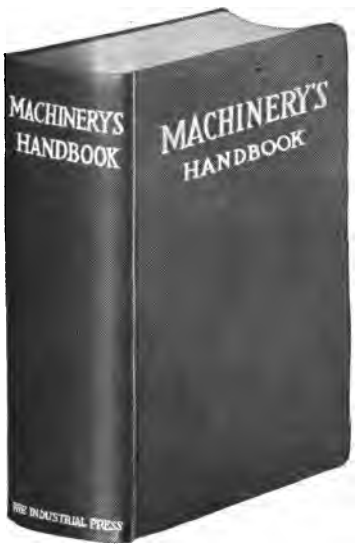

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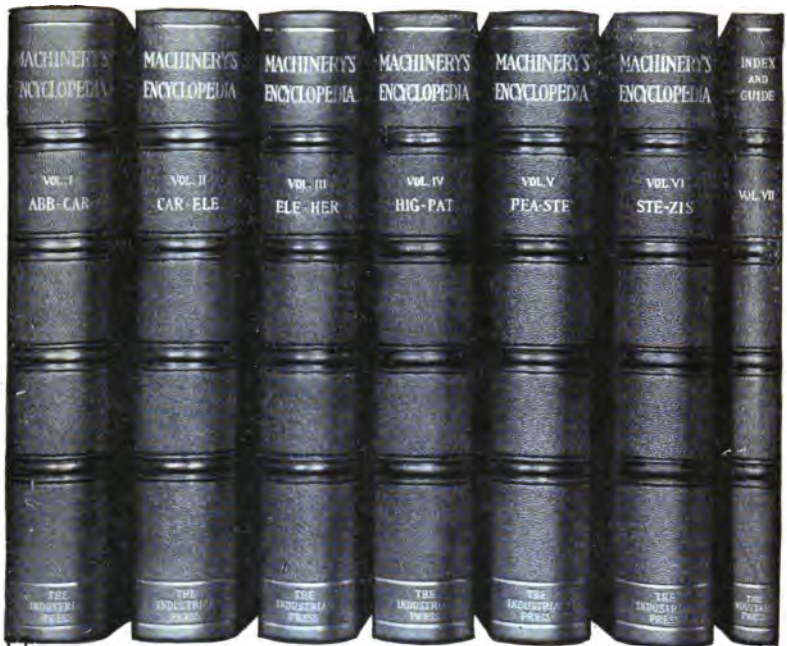

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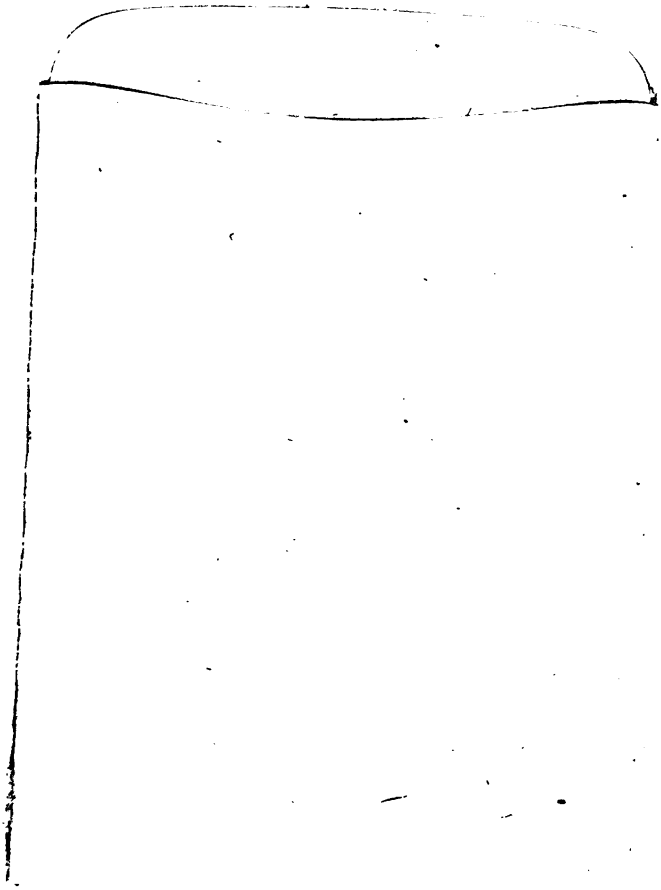
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